THE COLLAPSE OF IRON-OXYGEN STARS: PHYSICAL AND MATHEMATICAL FORMULATION OF THE PROBLEM AND COMPUTATIONAL METHOD

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Abstract. A statement of the problem of gravitational collapse and a computational method are described. The main feature of the collapse — its extremely high heterogeneity — is taken into account. The structure of a collapsing star is characterized by a dense and hot nucleon core which is opaque with respect to neutrino radiation and is embedded into an extended envelope, almost transparent to neutrinos. The envelope is gradually being accreted onto the core. The enormous amount of energy, radiated in the form of neutrinos and antineutrinos, make us pay particular attention to relatively small absorption of neutrino radiation by extended envelope (so-called energy of deposition). The inclusion of the energy deposition in the calculations is of importance for the problem of transformation of an implosion into an explosion. The deposition is taken into consideration in the approximation of diluted neutrino radiation which escapes from neutrino photosphere and is partially absorbed in the envelope. Both the generation of energy due to deposition and the change of neutron-to-proton ratio are taken into account. The increase of the mass of the core, which is opaque with respect to neutrino radiation, is fully taken into account in the calculations of the gravitational collapse.

1. Introduction

Colgate and White (1966) were the first to take account of the neutrino opacity in the dynamics of the gravitational collapse. As a result, the problem of energy transport from collapsing stellar core to stellar envelope with the aid of neutrinos has been put forward (so-called deposition problem). It was believed that, under favourable conditions, this energy transport could either induce the ejection of an envelope, observed as supernova outburst (pure deposition), or stimulate a flash of unburnt nuclear fuel (oxygen) in stellar envelope (deposition as a triggering mechanism in the Fowler’s and Hoyle’s (1964) supernova model). The subsequent investigations (Arnett, 1966, 1967; Ivanova et al., 1969) have indicated that the deposition problem is indeed a very complex one, both in the physical and mathematical sense. We have no possibility to go here into details (one can refer to Zel’dovich’s and Novikov’s book (1971) and to an article by Imshennik and Nadyozhin (1974) presented at Copernicus’s Assembly of IAU).

It is worthwhile to note that the main difficulty in the solution of this problem consists in the correct treatment of neutrino interaction with those layers of the collapsing core which are either semitransparent or opaque to neutrino radiation. The straightforward way of investigation of the neutrino deposition problem involves the numerical solution of hydrodynamical equations together with the equation of
neutrino transfer (Wilson, 1971). However, a number of inevitable simplifications (very crude physical approximation, inadequate size of difference, mesh, etc.), which arise from the extreme complexity of the problem, bring the calculations of this kind at best to a status of numerical experiment.

A different approach is utilized in the present paper. It deals with the use of two asymptotic solutions of the equation of transfer: one solution being valid inside the collapsing core, which is highly opaque to neutrino radiation, and the other is applicable to neutrino interaction with the nearly transparent stellar envelope. The transfer of energy and lepton charge by means of neutrinos and antineutrinos inside the collapsing stellar core after onset of neutrino opacity is considered here in the neutrino thermal conductivity approximation (Imshennik and Nadyozhin, 1972). The gravitational collapse proves to be highly heterogeneous. Therefore, an intermediate layer between the opaque stellar core and transparent envelope contains a small fraction of stellar mass. This property of the collapse facilitates considerably the fitting of the envelope to the core in the course of computations.

In the present paper we consider the basic equations and the main methodical features of the calculations which were fulfilled for rather massive iron-oxygen stars. The astrophysical side of these calculations will be discussed in the next paper.

2. Initial Models and Differential Equations of the Problem

The initial hydrostatically equilibrium model can be conveniently represented in the form of a gaseous sphere with a polytropic index $n=3$ (Ivanova et al., 1969). It will hereafter be assumed that the interiors of the initial model consists of iron-group nuclei and its envelope is composed of unburnt nuclear fuel (oxygen). The boundary between the iron core and oxygen envelope is chosen as close to the stellar centre as possible in order to meet the requirement of oxygen being unburnt up to the beginning of the collapse. This model gives the maximal efficiency of oxygen burning against a background of the collapse. Indeed, oxygen in this case is in the closest vicinity of the iron core – i.e., under the most favourable conditions for detonation. The stellar structure-like adopted initial model could be formed either as a result of large-scale mixing at the advanced stages of stellar evolution or due to intensive outflow of hydrogen-helium envelope, followed by an extinction of silicon and oxygen burning shells. However, the investigation of the evolution of carbon-oxygen stars (see, for example, Ikeuchi et al., 1971, 1972) within the framework of classical spherically-symmetric theory without mass loss and large-scale mixing (due to meridional circulation, for example) indicates that the forming iron core to be separated from massive envelope by one or two burning shells. The stellar models, obtained in these calculations, have rather a giant-like structure just before the onset of dynamical instability. Therefore, the main storage of unburnt nuclear fuel is settled down at the radii which are considerably greater than in our simplified model.

For a specified mass of initial model, the single arbitrary parameter is the radius of