THE SECULAR VARIATIONS IN THE RESTRICTED PROBLEM'S SIXTH-ORDER THEORY AND STABILITY: AN EXPLICIT NUMERICAL APPROACH

CATHERINE J. FLOGAITIS, V. S. GEROLYANNIS, and G. A. ANTONACOPOULOS
Astronomy Laboratory, Department of Physics, University of Patras, Greece

(Received 23 October, 1989)

Abstract. In this paper, we develop and implement an explicit numerical technique for studying equilibrium solutions concerning the secular variations in the restricted problem of three bodies, and their stability. In this implementation, we employ a sixth-order theory for the secular terms.

1. Introduction

In a previous paper (Geroyannis and Flogaitis, 1989; hereafter referred to as Paper I) we developed an explicit numerical method for studying the secular variations in the restricted problem of three bodies. This method consists in a systematic scanning of the \( \alpha - H \) plane, where \( \alpha \) denotes the semi-major axis and \( H \) stands for the Delaunay variable. By such an explicit scanning, the equilibrium solutions, assigned to a particular point \( (\alpha, H) \) of the \( \alpha - H \) plane, are localized as roots of a system of two highly nonlinear algebraic equations (Paper I, Equations (2.62a, b)). These equations result from the requirement that the partial derivatives of the modified Hamiltonian \( F^*(\alpha^{1/2}, H; p, q) \) in the two fundamental variables \( p \) and \( q \) (Paper I, Equations (2.19), (2.14a), and (2.14b), respectively) must be equal to zero in order for the roots \( \bar{p} \) and \( \bar{q} \) to represent equilibrium solutions. In addition, the roots \( \bar{p} \) and \( \bar{q} \), evaluated by a rootfinding algorithm, must satisfy four necessary conditions (Paper I, Section 3, Equations (3.7)-(3.9)). For a pure numerical-analysis viewpoint, root estimates \( \bar{p} \) and \( \bar{q} \) are satisfying these conditions are 'Acceptable Equilibrium Solutions', abbreviated AES, for the equations of motion (Paper I, Equations (2.61a, b)), and denoted by \( \bar{p} \) and \( \bar{q} \).

However, the problem of distinguishing 'stable' and 'unstable' AES remained open. The stability analysis, required for the higher-order terms of our equations, seems to present great difficulties when viewed according to Liapounov's methodology (Liapounov, 1907). Therefore, in order to study stability in the full range of terms involved in our fourth-order expansions, we have developed and implemented in a recent paper (Geroyannis and Flogaitis, 1990; hereafter referred to as Paper II) an 'Explicit Numerical Stability-Analysis Technique', abbreviated ENSAT, which fits very well the framework of the numerical strategy of Paper I. Moreover, ENSAT seems to be a general technique, applicable to other similar stability problems.

In the present paper, we shall use sixth-order expansions in studying both equilibrium
solutions and stability. For clarity and convenience, we shall use hereafter the definitions and symbols of Papers I and II.

2. Sixth-Order Theory

By retaining terms up to sixth-order in the expansions for the secular variations, we find

![Graph of acceptable equilibrium solutions, AES, on the α-H plane.](image-url)