ROTATIONALLY-PERTURBED ORBITAL ELEMENTS OF CLOSE BINARY SYSTEMS

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Abstract. The aim of this investigation is to present the secular and periodic perturbations of the six orbital elements of a close binary system due to rotational distortion. In our study we consider very small inclinations \( i \) of the orbital plane of the system, whereas the eccentricity of the orbit may assume any value between \( 0 < e < 1 \). The final formulae for the various elements have been expressed by means of the unperturbed true anomaly measured from the ascending node.

1. Introduction

Close binary systems are associations of two stars revolving around their common centre of mass. The size of each component represents an appreciable fraction of their separation. Such systems are bound to be distorted by axial rotation and mutual tidal action. The magnitude of the distortion is increasing with decreasing distance of the two stars.

Doppler shifts observed in the spectra of the components of close binary systems are not true indicators of the radial velocities of their absolute space motion. Only if the stars are spherical and the distribution of surface brightness over their apparent discs is characterized by radial symmetry, the observed Doppler shifts are indeed directly indicative of the radial velocity of the mass centre of the respective star, due to its Keplerian motion. Distorted close binary systems due to rotation and tides, will not only cease to be radially symmetrical, but the shape of the isophotes over their apparent discs will vary with phase; this phenomenon is furthermore augmented by the effects of mutual irradiation, even if the components could be regarded as spheres.

All these effects are bound to make the observed radial velocities of the components of a close binary system deviate from the Keplerian motion of their mass centre. Terms arising from their axial rotation will superpose upon the purely orbital velocity to yield a resultant which — if analysed in the classical manner, without regard to these complications — would furnish elements of spectroscopic orbits which are systematically in error.

Observations have shown, that in detached binary systems with circular orbits, there is synchronization between rotation and revolution (Olson, 1968; Nairai, 1971). In systems with eccentric orbits the stars rotate faster than the average velocity of their

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revolution (Swings, 1936). If the equator of each component is not co-planar to the orbit, dissipative forces secularly tend to rectify these axes and make them perpendicular to the orbit.

Secular perturbations of the orbital elements of close binary systems in the plane of the orbit (i.e., the semi-major axis $a$, eccentricity $e$, and longitude of the periastron $\omega = \Omega + \omega$) arising from rotational distortion have been presented in a paper by Kopal (1972). A more general treatment to include the perturbation of all the six elements of the orbit and the contribution of the periodic terms is given in Zafiropoulos and Kopal (1982) (hereafter referred to as Paper I).

In the present paper we propose to set up a simultaneous system of variational equations governing the perturbations of the six elements of the orbit. The components of the disturbing accelerations due to rotation, in the case of very small inclinations of the orbital plane, will be substituted in the above equations to yield the first-order approximation.

### 2. Equations of the Problem

Consider an isolated close binary system consisting of two components of constant masses $m_{1,2}$. We adopt the center of gravity of one of them (say $m_1$) as the origin of the inertial system of rectangular coordinates $x, y, z$. The differential equations governing the relative motion of the component of mass $m_2$ around $m_1$ at time $t$ are known to be of the form

\[
\frac{d^2 x}{dt^2} + \frac{G(m_1 + m_2)}{r^3} \frac{x}{r^3} = \frac{\partial R_{12}}{\partial x}, \tag{1}
\]

\[
\frac{d^2 y}{dt^2} + \frac{G(m_1 + m_2)}{r^3} \frac{y}{r^3} = \frac{\partial R_{12}}{\partial y}, \tag{2}
\]

\[
\frac{d^2 z}{dt^2} + \frac{G(m_1 + m_2)}{r^3} \frac{z}{r^3} = \frac{\partial R_{12}}{\partial z}, \tag{3}
\]

where

\[ r^2 = x^2 + y^2 + z^2 \]  

is the radius-vector of the relative orbit, $G$ denotes the constant of gravitation and $R_{12}$ represent the 'disturbing function' arising from the departure of both components from spherical symmetry because of axial rotation.

Equations (1) to (3) constitute a simultaneous differential system of 3 second-order differential equations. This system can be solved in a closed form, only in the absence of a disturbing function. The solution introduces six integration constants, known as the elements of the orbit. These elements are,

- $\Omega$ = longitude of the ascending node;
- $i$ = inclination of the orbital plane;