DIRECT PHOTOMETRIC PROBLEM FOR SPOTTED STARS

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Abstract. In the framework of the direct photometric problem we obtained an analytical representation of the light variations of binary systems of stars one of which with spots on its surface. The photometric effect is due to the mutual eclipses and to the rotation of the spotted component. These expressions can be used for the solution of the inverse problem for spotted stars.

1. Introduction

Recently, the explanation of the cyclic light variations of more and more astronomical objects the spot hypothesis has been applied. The methods so far developed to obtain information for the physical parameters of the spots (location, temperature, size) from the observational light curves are based on numerical computations (cf. Poe and Eaton, 1985; Vogt, 1981). They give only the limits for the values of these parameters.

We consider that, before we proceed to the solution of such an inverse problem we must solve analytically the corresponding direct problem -- i.e., to obtain the light variations of stars with given spot configuration and, on the basis of such considerations, to build the solution of the inverse problem.

Since the spotted stars may be members of binary systems, the photometric effect is due to the mutual eclipses and to the rotation of the spotted star. Therefore, in the framework of the direct photometric problem, we shall analyze the light variations of binary systems of stars one of which with spots on its surface.

2. The Problem

We consider a contact system (the generalization for detached systems is simple) of stars with radii \( R_1 \) and \( R_2 \), whose orbital plane is inclined under angle \( I \) with regard to the line-of-sight. On the surface of the former there is a circle spot with radius \( r_0 \) in a polar distance \( \beta \). The spot temperature differs from the temperature of the surrounding photosphere (the generalization for stars with more spots can be done by summation of the effects of the individual spots). The spotted star rotates with angular speed \( \omega \) around its rotational axis concluding an angle \( i \) with the line of sight and an angle \( \zeta \) with the orbital plane normal. Besides it moves on the secondary star in a circular orbit with an angular velocity \( \Omega \).

We adopt a linear laws for limb-darkening of the stars and of the spot

\[
I_k(\theta) = I_{0k}(1 - u_k + u_k \cos \theta), \quad k = 1, 2
\]

\[
I^{sp}(\theta) = I_0^{sp}(1 - u^{sp} + u^{sp} \cos \theta).
\]

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At every moment $t$ the total light of the system is given by the following sum

$$B(t) = B_1 + B_2 + \mu B^{sp}(t) - B^{oc}(t) - \mu B^{sp}_{\#}(t),$$

where $B_1$ and $B_2$ are the lights of the primary (without spot) and of the secondary component respectively; $B^{sp}(t)$ is the contribution of the spot; $B^{oc}(t)$ is the light of the covered area of the eclipsed component at the moment $t$; $B^{sp}_{\#}(t)$ is the light of the covered part of the spot from the secondary component and

$$\mu = \begin{cases} 1, & \text{when the spot is visible,} \\ 2, & \text{when the spot is invisible.} \end{cases}$$

In Equation (2):

$$B_k = \pi R_k^2 I_{0k}(1 - u_k/3), \quad k = 1, 2$$

and for $B^{sp}(t)$ the following expression

$$B^{sp}(t) = \pi R_0^2 \left[ I_0^{sp}(1 - u^{sp}) - I_{01}(1 - u_1) \right] (\cos \beta \cos i + \sin \beta \sin i \cos \omega t) +$$

$$+ \frac{\pi R_1^2}{3} (I_0^{sp} u^{sp} - I_{01} u_1) \left\{ -2 - \frac{r_0^2}{R_1^2} \sqrt{1 - \frac{r_0^2}{R_1^2}} +$$

$$+ 3 \frac{r_0^2}{R_1^2} \sqrt{1 - \frac{r_0^2}{R_1^2}} (\cos \beta \cos i + \sin \beta \sin i \cos \omega t)^2 \right\}$$

is known (cf. Kjurkchieva and Shkodrov, 1986).

Thus the problem of the construction of the light curve of a binary system one of which component has spots on its surface reduces to the calculation of $B^{oc}(t)$ and $B^{sp}_{\#}(t)$.

3. The Solution

In the intervals

$$\alpha \leq \Omega t \leq \pi - \alpha \quad \text{and} \quad \pi + \alpha \leq \Omega t \leq 2\pi - \alpha,$$

where

$$\alpha = \arcsin \frac{R_j}{R_j + R_k}, \quad j \neq k, \quad j, k = 1, 2,$$

the light of the eclipsed area is given by the expression

$$B^{oc}_{\#}(t) = \int_{R_j}^{R_k} I_{0k} \left( 1 - u_k + u_k \frac{\sqrt{R_k^2 - r^2}}{R_k} \right) 2\gamma_{\#} r \, dr,$$

where $2\gamma_{\#}$ is the covered arc of the isoline with radius $r$ at the eclipse. From geometrical