Abstract. We find that the Chandrasekhar–Kendall functions \( A \) do not satisfy the identity
\[
(A \cdot \nabla) A = (\nabla \times A) \times A + \frac{1}{2} \nabla A^2
\]
and, therefore, the results, in magnetohydrodynamics and other fields, obtained by two independent authors may differ from one another.

The Chandrasekhar–Kendall functions (C–K functions), the eigenfunctions of a curl operator (Chandrasekhar and Kendall, 1957), have been used by several authors. For example, Montgomery et al. (1978) and Krishan (1983a, b, 1984) have expressed the magnetic field \( \mathbf{B} \) and fluid velocity \( \mathbf{V} \) in terms of C–K functions as
\[
\mathbf{B} = \sum_{n,m,q} C_{nmq} \lambda_{nmq} \xi_{nmq} A(n, m, q),
\]
\[
\mathbf{V} = \sum_{n,m,q} C_{nmq} \lambda_{nmq} \eta_{nmq} A(n, m, q),
\]
where \( \xi_{nmq} \) and \( \eta_{nmq} \) are expansion coefficients, \( C_{nmq} \) is the normalization factor, and \( A(n, m, q) \) is the C–K function given by (in cylindrical coordinates \( r, \theta, z \))
\[
A(n, m, q) = \psi(n, m, q) = J_n(\gamma_{nmq}, r) \exp(im\theta + ik_nz),
\]
where
\[
\psi(n, m, q) = J_m(\gamma_{nmq} r) \exp(im\theta + ik_nz),
\]
\[
\gamma_{nmq}^2 = \gamma_{nmq}^2 + k_n^2, \quad k_n = 2\pi n/L,
\]
\[n = 0, \pm 1, \pm 2, \pm 3, \ldots, \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots, \quad \gamma_{nmq} > 0;\]
\( \lambda_{nmq} \) being the eigenvalue corresponding to the eigenfunction \( A(n, m, q) \) [i.e., \( \nabla \times A = \lambda A \)], and \( L \), the length of the cylinder and \( J_n(\gamma r) \) is the Bessel function.

A well-known relation in the vector calculus asserts that
\[
\nabla(\mathbf{P} \cdot \mathbf{Q}) = (\mathbf{P} \cdot \nabla) \mathbf{Q} + (\mathbf{Q} \cdot \nabla) \mathbf{P} + \mathbf{P} \times (\nabla \times \mathbf{Q}) + \mathbf{Q} \times (\nabla \times \mathbf{P}),
\]
which for \( \mathbf{P} = \mathbf{Q} \) may be rearranged as
\[
(\mathbf{P} \cdot \nabla) = \mathbf{P} = (\nabla \times \mathbf{P}) \times \mathbf{P} + \frac{1}{2} \nabla \mathbf{P}^2.
\]
Now if \( P \) is a C–K function, then the relation (4) reduces to

\[
(A \cdot \nabla)A = \frac{1}{2} \nabla A^2 .
\]  

(5)

However, we find that the relation (5) is not satisfied by the C–K functions. To show it explicitly, let us consider a C–K function with \( m = n = 0 \):

\[
A(0, 0, q) = \gamma J_1(\gamma r) \hat{e}_\theta + \gamma J_0(\gamma r) \hat{e}_z .
\]

Now

\[
A \cdot \nabla = \gamma \left[ J_1(\gamma r) \frac{\partial}{\partial \theta} + J_0(\gamma r) \frac{\partial}{\partial z} \right]
\]

and

\[
(A \cdot \nabla)A = 0 .
\]

However,

\[
A^2 = \gamma^2 [J_1^2(\gamma r) + J_0^2(\gamma r)]
\]

and

\[
\frac{1}{2} \nabla A^2 = - \frac{\gamma^2}{r} J_1^2(\gamma r) \hat{e}_r .
\]

Hence,

\[
(A \cdot \nabla)A \neq \frac{1}{2} \nabla A^2 .
\]  

(6)

This situation may effect in several scientific problems, one such case is discussed below: for the plasma in a cylinder of radius \( R \) and length \( L \) with regid perfectly conducting walls at \( r = R \), the magnetohydrodynamical equations are

\[
\rho \left[ \frac{\partial V}{\partial t} + (V \cdot \nabla) V \right] = \frac{1}{4 \pi} (\nabla \times B) \times B - \nabla p + \rho g ,
\]

(7a)

\[
\frac{\partial B}{\partial t} = \nabla \times (V \times B) ,
\]

(7b)

\[
\nabla \cdot B = \nabla \cdot V = 0 ;
\]

(7c)

where \( \rho \) is the mass density and \( p \) is the pressure in the region. In many cases, the length of the cylinder is less than the scale-length and, therefore, the effect of gravity (\( g \)) may be neglected. Expressing the magnetic field and fluid velocity by a single C–K function

\[
B = C_{nmq} \lambda_{nmq} \xi_{nmq} A(n, m, q) , \quad V = C_{nmq} \lambda_{nmq} \eta_{nmq} A(n, m, q) ,
\]

we have \( (\nabla \times B) \times B = 0 \) and the steady-state pressure gradient is given by

\[
\nabla p = -\rho (V \cdot \nabla) V ,
\]

(8)