PARTICLE ESCAPE PROBABILITY FOR INHOMOGENEOUS MEDIUM

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(Received 21 November, 1986)

Abstract. The particle escape probability from a non-multiplying slab containing an internal source is defined in terms of a functional relation for the scattering function for the diffuse reflection problem. The Padé approximation technique is used to obtain numerical results for particle escape probability for inhomogeneous medium. Numerical results for homogeneous and inhomogeneous media are given.

1. Introduction

A method is proposed to express the flux emerging from an inhomogeneous or homogeneous medium in terms of the emergent flux of the diffuse problem. This relation between the two problems permit to define the escape probability for non-multiplying medium in terms of the source function of the diffuse problem. The escape probability in this way is defined for arbitrary internal source. The Padé approximant technique is used to calculate the functional relations define the escape probability. Numerical results are given for [0/1] Padé approximant for homogeneous and inhomogeneous media with uniform isotropic source. The results for homogeneous medium are compared with the results of Pomraning and Badaham (1984).

2. Basic Equations

Consider the particle transfer equation in a non-multiplying slab \( 0 \leq x \leq a \) with internal source

\[
\left( \mu \frac{\partial}{\partial x} + 1 \right) I(x, \mu) = \frac{\lambda(x)}{2} \int_{-1}^{+1} I(x, \mu') d\mu' + (1 - \lambda(x))g(x) = S(x),
\]

subject to the boundary conditions

\[
I(0, \mu) = 0 , \quad \mu > 0
\]

and

\[
I(a, \mu) = 0 , \quad \mu \leq 0.
\]
where distances are measured in mean-free paths. The source and the scattering are assumed to be isotropic, $\lambda(x)$ is the single-scattering albedo, i.e., the number of secondaries emitted per collision, $\mu \in [-1, 1]$ is the cosine of scattering angle.

The integral equation for the source function

$$S(x) = \frac{\lambda(x)}{2} \int_0^a E_1(|x - x'|)S(x') \, dx' + (1 - \lambda(x))g(x),$$  \hspace{1cm} (4)

where

$$E_n(x) = \int_0^1 u^{n-2} \exp(-x/u) \, du$$  \hspace{1cm} (5)

is the exponential integral.

If the source function $S(x)$ is found, the intensities emerging from the slab are

$$I(0, -\mu) = \frac{1}{\mu} \int_0^a S(x) \exp(-x/\mu) \, dx, \quad \mu > 0$$  \hspace{1cm} (6)

and

$$I(a, \mu) = \frac{1}{\mu} \exp(-a/\mu) \int_0^a S(x) \exp(x/\mu) \, dx.$$  \hspace{1cm} (7)

The escape probability is defined in terms of the emerging flux by

$$P = \frac{H(0) + H(a)}{4\pi \int_0^a [1 - \lambda(x)]g(x) \, dx},$$  \hspace{1cm} (8)

where

$$H(0) = 2\pi \int_0^1 \mu I(0, -\mu) \, d\mu$$  \hspace{1cm} (9)

and

$$H(a) = 2\pi \int_0^1 \mu I(a, \mu) \, d\mu;$$  \hspace{1cm} (10)

and from Equations (6) and (7) one gets

$$H(0) = 2\pi \int_0^a S(x)E_2(x) \, dx.$$  \hspace{1cm} (11)