FOURIER ANALYSIS OF THE LIGHT CURVES OF ECLIPSING VARIABLES, XIV*

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Abstract. The aim of the present paper will be to utilize the results obtained in the preceding papers of this series for the development of practical procedures for obtaining the elements of any eclipsing system from the observed photometric data by their analysis in the frequency-domain, for any type of eclipses, any proximity of the two components, and any degree of the law of limb-darkening of the eclipsed star.

In Section 2, which follows a brief introduction to the subject, procedures will be developed which should permit us to perform such an analysis – by hand or automatic machine computation – for the case of mutual eclipses in binary systems the components of which can be regarded as spheres; and whose apparent discs are characterized by an arbitrary radially symmetrical distribution of surface brightness. In Section 3 we shall generalize these procedures to systems consisting of arbitrarily distorted stars.

1. Introduction

In the preceding papers of this series (Kopal, 1975a, b, c, d, e; 1976a, b; 1977a, b, c) a new approach has been made to the problem of an analysis of the light changes of eclipsing binary systems in the frequency-domain; and the principal advantages of this novel approach pointed out: namely, (a) the possibility of a separation of the proximity effects (of any magnitude) from those caused by eclipses by purely algebraic means; and (b) the existence of algebraic relations between the observed characteristics of the light curves and the unknown elements of the eclipses, which can be solved for the latter by convenient numerical methods. The aim of the present paper will be to summarize the gist of our preceding analysis in so far as it bears directly on the practical aspects of our central problem, and to illustrate such procedures by application to representative cases of actual eclipsing systems – both close and wide.

After having brought – in four years of continuous effort – the analysis of the light curves of eclipsing variables in the frequency-domain to its present stage, at which all difficulties met on the way have at last been successfully prostrated, the senior writer feels almost obliged to apologize to the readers of his earlier papers for so much previous ‘beating about the bush’, and apparent complexity of certain parts of his earlier approach to the problem, which eventually proved to have been mere detours on the way towards its optimum solution as presented in this paper. In fact,

* Paper dedicated to Professor Hannes Alfvén on the occasion of his 70th birthday, 30 May, 1978.
the problem did not appear to us in its stark simplicity, and the *royal road* to its
solution was not reached until 1976 (see Paper XI, Kopal, 1977b), when the fractional
loss of light during eclipses was identified with a cross-correlation of two 'apertures'
which represented the eclipsing and eclipsed components; and this feat necessitated
an excursion into the frequency-domain to evaluate the convolution integral of a
product of the Fourier transforms of the two apertures. Once this was done, all
subsequent developments became almost automatic and led rapidly to our goal.

Although some of our earlier digressions – for instance, an exploration of the role
of quadratic moments of the light curves – turned out of only limited significance as
far as the practical aims of our work are concerned, their outcome continues to be
of theoretical interest, and their exploration should also reassure us that no sidelines
were left in the dark, or no stones unturned on the way. Moreover, repeated refer-
ences to results previously established in the preceding papers of this series should
facilitate our aim to present our results in a concise form geared primarily towards
applications: only the most essential results previously proven will be reproduced
below for the sake of completeness; and to facilitate reading, our notations (barring
exceptions noted in the text) will likewise be made strictly consistent with those
previously employed by us.

2. Systems Consisting of Spherical Stars

The aim of the present section will be to detail practical methods by which an analysis
of the light curves for the elements of eclipsing systems which consist of spherical
stars can be performed in the frequency-domain for any type of eclipses. At first sight,
the adoption of spherical shape for both components of such systems may seem to
be unduly restrictive, and limit the applicability of a model based upon it only to a
very small class of systems which are sufficiently wide for mutual distortion of both
stars to be negligible.

However, by virtue of their relatively low probability of discovery (narrow eclipses!) alone, such systems are likely to constitute but a very small fraction of eclipsing
systems known to us at the present time. As the probability of their discovery increases
with increasing proximity of their components, so does their mutual distortion; and
the proximity effects arising from it are bound to cease to be negligible. However, as
will be shown in Section 3, a solution for the elements of distorted eclipsing systems
can always be reduced to one based on a spherical model. This fact should make the
subject matter of the present section fundamental for an analysis of *any* light curve
in the frequency-domain – regardless of whether or not the form of the components
whose mutual eclipses give rise to the observed light changes is spherical or distorted:
in the former case, it represents the final solution; in the latter, a necessary prerequisite
for subsequent developments.

As has been pointed out already in Paper I (Kopal, 1975a), the fundamental ob-
served quantities which will serve as a basis for a determination of the elements of