RELATIVISTIC DYNAMICS OF EXPANDING SOURCES

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Abstract. Given a relativistically expanding source, the transformation of the emission spectrum from the comoving frame of the source to the observer's frame is calculated. In particular, we deal with two special cases, that of a uniformly expanding sphere and that of a cylinder. In each case we consider a power-law spectrum with two distinct regions - the optically thick and thin parts - and transform those regions separately. The results are discussed, and a generalized procedure for transforming a spectrum of unspecified form is outlined. This procedure takes into account any bulk motion by the source as well as expansion. Finally, applications of this procedure to astrophysical sources are briefly discussed.

1. Introduction

Typical non-thermal processes in astrophysics involve a power-law energy spectrum of electrons. Consequently, the emission spectrum shows a power-law region at high frequencies, and also shows a self-absorption region at low frequencies, where the source is optically thick. When such a source has bulk relativistic motion, the transformation from the frame of the source to that of the observer is simple enough - the Doppler shift of a power-law remains a power-law of the same spectral index.

In this work we have calculated the transformations necessary for a relativistically expanding source. For such a source, the transformation from the comoving frame to the laboratory frame is complicated by the fact that the low-frequency, self-absorption part of the spectrum must be treated differently from the high-frequency, optically thin part of the spectrum. In the former case only the front surface is involved in the emission process, but in the latter case the entire volume contributes to the emission.

In this paper we deal with two special cases, that of a uniformly expanding sphere and that of a cylinder. We consider these cases for their ease in calculation, and as prototypes for generalization to other morphologies. We also consider them useful in modeling a host of astrophysical phenomena. Cylinders approximate well the morphology and dynamics of extragalactic jets, for example.

We also calculate the transformations for a source that is both expanding and has bulk motion. Our procedures are general and may be applied to any spectrum, but a detailed study for power-law emission has been made. We have taken into account the Doppler shift in frequency and the effects of beaming. The results are analyzed, and applications to astrophysical sources are discussed.

Other authors (see Rees, 1967; Ozernoy and Sazonov, 1968, 1969) have taken similar initial conditions to investigate the emission of specifically synchrotron processes, and have considered how such emission changes in time with changing geometrical and physical conditions.

The work of Königl (1978), for instance, involves the flux variability (in time) due to a relativistically expanding source, as seen at a particular frequency. While Königl's calculation is constant in frequency and variable in time, here we treat time as a constant and frequency as the variable. Königl's work facilitates the interpretation of observations made at a particular frequency over a range of times, while this work is intended to facilitate the interpretation of observations covering a range of frequencies taken at a particular time.

2. General Description

We begin with a relativistically expanding source in which every differential volume element is emitting radiation as a power-law as seen in its own comoving rest frame. This is common among non-thermal, relativistic systems such as, for instance, the synchrotron process. In such circumstances, there is a frequency below which the radiation is self-absorbed. This depends on the density of the medium as it expands, so the turnover frequency shifts gradually with the expansion to lower frequencies (see Sadun, 1988; van der Laan, 1966). Above that frequency, each element of volume is emitting as a power-law spectrum; the radiation travels essentially unimpeded throughout the volume of the source to the observer. Therefore, we must concern ourselves with the transformations of two separate regions of the spectrum, namely the optically thick and the optically thin regions.

This demarcation between the self-absorbed and transparent parts of the spectrum corresponds as well to the physical dimension with which we need to deal. For the optically thin part of the spectrum, the radiation the observer receives is being emitted from the entire volume of the source, which includes parts that are expanding away from him (and are correspondingly redshifted). For the optically thick part of the spectrum, the radiation the observer receives is being emitted from the front surface of the source only, and so contains primarily blueshifted components of the emission.

In performing the Lorentz transformations from the comoving frame to the observer's frame for each of the two cases (optically thin and thick), we need to consider the two phenomena related to relativistic dynamics. The radiation will be redshifted according to the motion (toward or away from the observer) of each volume (or surface) element, and the radiation will be beamed preferentially in the direction of its motion. Thus elements of the source moving toward the observer will be blueshifted and beamed strongly toward the observer, while the elements moving away from the observer will be redshifted, but will contribute less to the overall spectrum because it is being beamed away from the observer. Finally, all the elements with their transformations are combined to yield the transformed spectrum in the observer's frame. Naturally, the degree to which these relativistic effects take place is strongly a function of the velocity of each radiating element relative to the observer.