THE CRITICAL AND THE SATURATION CONTENT OF MAGNETIC MONOPOLES IN ROTATING RELATIVISTIC OBJECTS*

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Abstract. Both the critical content \( \zeta_c \) (\( \zeta_c \equiv N_m/N_B \), where \( N_m, N_B \) are the total numbers of monopoles and nucleons, respectively, contained in the object), and the saturation content \( \zeta_s \) of monopoles in a rotating relativistic object are found in this paper. The results are:

\[
\zeta_c = \zeta_{c0} \left(1 - 4\alpha^2/R_g^2\right)^{1/2}, \quad \zeta_{c0} \equiv \frac{Gm_B/g_m}{g_m} = 4.365 \times 10^{-21},
\]

where \( \alpha \) is the specific angular momentum of the object; \( R_g \), the Schwarzschild radius of the object; \( g_m \), the magnetic charge of a stable colourless monopole \( g_m = 3hc/4\pi e \).

(2) For a non-rotating object (\( \alpha = 0 \)),

\[
\zeta_s = \zeta_n \left(1 - R_g/R\right)^{-1/2}
\]

when

\[
(R/R_g)^2 \geq 1 \quad \text{or} \quad \zeta_s = \sqrt{2} \beta^{-1/2} \sqrt{\frac{R}{R_g}} \zeta_n \quad \text{when} \quad R/R_g < 1 + \beta,
\]

where \( R \) is the radius of the object; \( \zeta_n \), the Newtonian saturation content of like monopole,

\[
\zeta_n = Gm_Bm_m/g_m^2 = 1.9 \times 10^{-25}(m_m/10^{16}m_p),
\]

\[
\beta = \zeta_n/\zeta_{c0} = 4.3 \times 10^{-5}(m_m/10^{16}m_p).
\]

Although the critical content cannot be reached, the induced nucleon decay by monopoles will prevent the massive objects (e.g., galactic nuclei and quasars) from collapsing into black holes (Peng et al., 1986a, b).

(3) For a rotating object, although the saturation content of monopoles is the same as above, the value of the critical content is greatly decreased for a fast rotating object. Due to the induced nucleon decay by monopoles, neither the horizon nor the central singularity exists for a collapsed object with \( R \lesssim \frac{1}{2}R_g \) which is rotating so fast that the condition \( \alpha > GM/c^2\left[1 - (\zeta_c/\zeta_{c0})^2\right]^{1/2} \) is satisfied. Those objects mainly radiate infrared radiation with rather strong \( \gamma \)-ray and X-ray.

1. Introduction

According to the inflational theory of the Universe, the entire observed Universe from a single initial fluctuation of Higgs field, the monopole overabundance problem which is caused by the smallness of the Higgs correlation length in the standard model disappears, but tiny amount of magnetic monopoles may yet be produced (Guth, 1982), due to the violent oscillation and thermal fluctuation of Higgs field during the phase transition of the initial universe which was very hot \( (kT > 10^{15} \text{ GeV}) \) and began in a highly chaotic state.

Peng et al. (1985) have estimated the saturation content of like monopoles in a Newtonian object: \( \zeta \) (\( \equiv N_m/N_B \), where \( N_m \) and \( N_B \) are the total number of monopoles

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and nucleons in the object, respectively), \[ \zeta_m = G m_B m_m / g_m^2 \approx 1.9 \times 10^{-25} \]

\[ (m_m/10^{16} m_B) (m_B \text{ and } m_m \text{ are the mass of a nucleon and a monopole, respectively, for a massive monopole of the 't Hooft-Polyakov-type, } m_m \approx 10^{16} m_B. g_m \text{ is the magnetic charge of a monopole, } g_m = 3hc/4\pi e = 9.88 \times 10^{-8} \text{ G for a stable colourless monopole (Ma and Tang, 1983), which is three times as large as Dirac magnetic charge, which is give orders of magnitude lower than the upper limit of the ratio of number of monopoles in the Universe, proposed by Parker (1970) and Lazarides et al. (1987), } \zeta \leq \zeta_0 \leq 10^{-20} \pm 1. \]

Assuming that the content of monopoles in primary clouds is \( \zeta_0 \), Peng (1988) re-estimated recently the amount of monopoles contained in a celestial object which was born from a primary cloud. The conclusions are: (a) The monopoles in both normal stars (e.g., the Sun) and planets (e.g., the Earth) are due mainly to capture after they formed and the content of monopoles is much less than Newtonian saturation content \( \zeta/\zeta_n \leq 10^{-12} \). (b) The monopoles in galactic nuclei and quasars are basically remained in the formation process of these objects, and the content of monopoles may be much higher than that in stars and in planets, \( \zeta/\zeta_0 \sim 10^{-3} (V_m/10^{-4} c)^{-3/2} M_1^{1/2} \), where \( M_12 \) is the mass of the object in \( 10^{12} M_\odot \) (mass of the Sun) unit, \( V_m \) is the velocity of monopoles in primary clouds in the early stage of the Universe and may be much less than that in the interstellar space today \( \sim 10^{-3} c \) (see Parker, 1970)), e.g., \( V_m \leq 10^{-4} c \) (see Lazarides et al., 1981). Hence, it is possible that the content of monopoles in some supermassive galactic nuclei and quasars may be near the Newton saturation, if \( \zeta_0 \sim \zeta_n \). The monopoles in white dwarfs and neutron stars are due mainly to capture and the the ratio is \[
\frac{\zeta}{\zeta_n} \leq \left(10^{-2} \sim 10^{-3}\right) \frac{R}{R_g} \frac{M}{M_\odot} \left(\frac{\langle \sigma \beta \rangle}{10^{-27} \text{ cm}^2}\right)^{-1} \frac{\tau}{10^{10} \text{ yr}},
\]

where \( R \) and \( M \) are the radius and the mass of the object, respectively; \( R_g \), its Schwarzschild radius; \( R_g = 2GM/c^2 \); \( \tau \), the age of the object; \( c\beta \), the velocity of a nucleon relative to a monopole. \( \langle \sigma \beta \rangle \) is the average cross-section of the catalyzed nucleon decay by monopoles \( pM \rightarrow Me^+ \pi^0 \) (85%) or \( Me^+ \mu^+ \mu^- \) (15%), which was proposed by Rubakov (1981, 1982, 1983) and Callan (1982a, b, 1983) (hereafter referred to as RC effect). But there is a great disparity in estimating the cross-section, \( \sigma \sim 10^{-25} \sim 10^{-26} \text{ cm}^2 \) by Rubakov and Callan or \( \sigma \sim 10^{-36} \text{ cm}^2 \) by Wilczek (1982). Hence, the content of monopoles in white dwarfs and neutron stars is lower than \( \zeta_n \) if \( \sigma \sim 10^{-26} \text{ cm}^2 \) or larger than \( \zeta_n \) if \( \sigma \sim 10^{-36} \text{ cm}^2 \).

Peng et al. (1986a, b) or Wang and Peng (1986) have suggested a model of galactic nuclei with monopoles under Newtonian mechanism and gave an explanation of the X-ray radiation from our galactic center to (Wang et al., 1985, 1986). We came to the conclusion that it is possible for the model of galactic nuclei to not be a black hole \((R/R_g > 1)\), so long as a parameter \( \xi = (\zeta/\zeta_n) (\langle \sigma \beta \rangle /10^{-27} \text{ cm}^2) \) is larger than \( 10^{-7} \). In our model, the huge power of galactic nuclei and quasars results from the RC effect and it may prevent these supermassive objects from collapsing to black holes. That is an interesting problem what would happen for a relativistic object with monopoles?