Abstract. We investigate the spherically symmetric, self-similar flow behind a blast wave from a point explosion in a medium whose density varies with distance as $r^{-\omega}$ with the assumption that the flow is isothermal and viscous. If $0 < \omega < \omega_c$ where $\omega_c = \frac{5}{3}[13-(160)^{1/2}]$ Lerche and Vasyliunas have shown in the inviscid situation that there exist two critical points in the flow speed-radial distance plane, and that all solutions are degenerate in that they pass through the lower critical point with the same slope. The present paper shows that as the viscosity tends to zero, the viscous flow does not tend towards the inviscid flow pattern. Now the validity of adiabatic blast wave models has elsewhere been shown to be questionable for supernova remnants, and the inviscid blast wave models have also been shown to be inappropriate for supernova remnants. Taken together with these previous results, the results of the present calculations strongly suggest that the assumption of isothermal blast wave behavior of supernova remnants, either viscous or inviscid is not valid. Since the adiabatic blast wave models have elsewhere been shown to be inappropriate descriptions of supernova remnants, it is doubtful whether the self-similar property can be invoked at all in the case of supernova remnants.

1. Introduction

The use of inviscid self-similar flows as vehicles for interpreting observations of supernovae remnants is well established. Recently such use has been called into question on several grounds: first, if the flow is initially assumed to be adiabatic, it has been shown (Solinger et al., 1975) that the neglect of the heat flow (the heart of the adiabatic approximation) is not permitted for parameters currently believed to obtain in supernovae remnants; second, if the flow is initially assumed to be isothermal it has been shown (Lerche and Vasyliunas, 1976) that the solutions are both linearly and non-linearly unstable, and further that the single fluid approximation is not, in general, valid for the same supernova parameters used in the adiabatic treatment.

As well as the questionable application of the inviscid self-similar flows to supernova remnants there are several fundamental problems of mathematical physics interest which have not yet been explored. It is a fact that no physical system can avoid having both a finite viscosity and a finite heat conductivity. They may be small so that their a priori neglect can be assumed, a calculation worked through, and then, a posteriori, the assumed neglect can be investigated for validity or invalidity. On at
least these grounds the self-similar flows used to describe supernovae remnants have been recently questioned.

The question then arises: What modifications must be made to the inviscid self-similar flow patterns when either viscosity or heat conduction are allowed for: This question is not completely academic for it is known (Lerche and Vasyliunas, 1976) that for inviscid spherically symmetric self-similar isothermal flow into a medium whose density varies away from the origin as \( r^{-\alpha} \) the flow pattern contains two singular points in \( 0 < \omega < \omega_c \).

The inviscid flow pattern has the following properties: All solutions of the first order inviscid flow equations pass through the lower critical point \textit{with the same slope}, only one solution leaving the lower critical point passes through the upper critical point (all other solutions skirt the upper critical point in a hyperbolic manner). It then proceeds to cross the blast front separating the self-similar flow from the external quiescent material.

We have remarked elsewhere (Lerche and Vasyliunas, 1976) – (hereafter referred to as LV) that this situation is degenerate in that the flow solution emanating from the origin can be imagined to join onto \textit{any one} of the solution curves entering the lower critical point from above. (See the more detailed arguments in LV.) Thus it can certainly be claimed that it matches smoothly onto the single solution which passes through the upper critical point and then proceeds onto the blast front. It can also be claimed that the solution from the origin matches smoothly onto \textit{any other solution}, none of which intersect the blast front and hence do not describe the physics.

We noted (LV) that this degeneracy of the solutions is, presumably, lifted by including say, viscosity. But we also remarked that it was unknown whether the inclusion of viscosity permitted sufficient freedom to enable the solution from the origin to indeed match smoothly onto the single solution going through the upper critical point and then on to the blast front. That is, it is not known whether inclusion of viscosity permits a solution or denies the possibility of any physically acceptable solution.

We shall concern ourselves with the resolution to this question for spherically symmetric isothermal self-similar flows.

### 2. The Self-Similar Equations including Kinematic Viscosity

#### A. REDUCTION TO THE SELF-SIMILAR FORM

The equations describing the conservations of mass and momentum are

\[
\frac{\partial \varrho}{\partial t} + \nabla (\varrho \mathbf{v}) = 0, \quad (1)
\]

\[
\varrho \left( \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right) = -\nabla p + \nu \nabla^2 \mathbf{v}, \quad (2)
\]

where \( \varrho, p, \mathbf{v} \) are the mass density, pressure and velocity respectively of the fluid, and \( \nu \) is the coefficient of kinematic viscosity which may be a function of time.