SPIRAL STRUCTURE AS A STANDING DENSITY-WAVE PACKET

R. MEINEL and G. RÜDIGER
Sternwarte Babelsberg, Potsdam, G.D.R.

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Abstract. Spiral density waves without radial group transport are discussed as a possible explanation of persistent global spiral patterns in galaxies without density-wave driving mechanism. The additional demand for a small winding-up rate favours two-armed patterns in most cases.

1. Introduction

The linear density wave theory of spiral structure (Lin and Shu, 1964; see also the review articles by Toomre, 1977; Lin and Lau, 1979; Bertin, 1980; Athanassoula, 1984), in its original simple formulation, is based on the assumption of small density perturbations of the form:

$$\delta \sigma = a(r) \cos \left( \omega t - m \varphi + \int k(r') \, dr' \right).$$

(1)

The stellar disk is described as infinitesimally thin; $r$ and $\varphi$ are the usual polar coordinates. Equation (1) represents a spiral wave with $m$ spiral arms and the angular velocity

$$\Omega_p = \frac{\omega}{m}$$

(2)

of the spiral pattern. The spirals are leading if $k > 0$, and trailing if $k < 0$ (with respect to the material angular velocity $\Omega(r) > 0$ of the equilibrium state of the disk).

If we apply the tight winding approximation

$$\frac{m}{|k|} \ll 1,$$

(3)

this leads to a simple local dispersion relation. For the fluid dynamical approach based on the Euler and continuity equations together with Poisson's equation for gravitation and a barotropic equation of state relating the two-dimensional pressure $p$ to the surface density $\sigma$ one obtains

$$(\omega - m \Omega)^2 = \kappa^2 + k^2 c_s^2 - 2\pi G \sigma_0 |k|.$$  

(4)

This equation is valid for sufficient distance from the corotation resonance $\Omega_p = \Omega$. The

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epicyclic frequency $\kappa$ is defined by

$$\kappa^2 = 2\Omega \left( 2\Omega + r \frac{d\Omega}{dr} \right)$$  \hspace{1cm} (5)

and $c_s$ is the velocity of sound defined by

$$c_s^2 = \frac{dp}{d\sigma_{\sigma = \sigma_0}};$$  \hspace{1cm} (6)

$G$ being Newton's gravitation constant and $\sigma_0$ is the surface mass density of the axisymmetric stationary equilibrium state. Apart from the Lindblad resonances $(\omega - m\Omega = \pm \kappa)$ this hydrodynamic result agrees well with the kinetic theory based on the collisionless Boltzmann equation. The value of $c_s$ is to be identified with the radial velocity dispersion of the stars. The parameters characterizing the equilibrium state, $\Omega$, $\sigma_0$, $c_s$ as well as the amplitude $a$ and wave number $k$ are assumed to be slowly varying functions of $r$ in the sense $|d\Omega/dr| \ll |k\Omega|$, etc. The right-hand side of Equation (4) is positive for all $k$ if the stability parameter

$$Q(r) = \frac{\kappa c_s}{\pi G \sigma_0} > 1.$$  \hspace{1cm} (7)

We assume this stability criterion (Toomre, 1964) is satisfied for all values of the radial distance $r$.

Under these conditions the solution (1) can be interpreted as a rigidly rotating global spiral pattern if the dispersion relation (4) is used to determine $k(r)$ for a given constant $\Omega_p = \omega/m$. In general there are four solution branches (short and long, leading and trailing waves). Choosing the short trailing solutions, $m = 2$ and suitable values for $\Omega_p$, first successful comparisons with observations were made (Lin and Shu, 1967).

The free wave solution (1), however, is no true global solution since it does not satisfy any realistic boundary conditions and is invalid at the resonances. The simplest approximation of correct global solutions is to consider wave packets built up by a superposition of free waves of the form (1) with frequencies $\omega \pm \Delta \omega$ around a mean value $\omega$ and corresponding wave numbers $k(r) \pm \Delta k(r)$. But such wave packets propagate radially with the group velocity

$$c_k = \frac{\partial \omega}{\partial k} = \frac{\text{sgn} k (\pi G \sigma_0 - |k| c_s^2)}{\omega - m\Omega}. $$  \hspace{1cm} (8)

This important fact was shown by Toomre (1969). Thus the persistency problem of spiral structure arose again and this led to a search for driving or amplifying mechanisms of spiral density waves. In particular the concept of unstable global modes was developed.

The aim of the present paper is to discuss the simpler possibility of interpreting the spiral structure as a wave packet with vanishing group velocity.