THE FERMI MECHANISM AND THE SOURCE SPECTRUM OF COSMIC RAY NUCLEI

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Abstract. It is shown that the velocity term, occurring in the expression for the rate of energy gain by the Fermi mechanism of acceleration, is to be taken into account in case of acceleration of non-relativistic particles. A spectral form of accelerated particles is derived on this basis and is called the 'Fermi Spectrum'. At non-relativistic energies this spectral form is significantly different from the currently used forms of power law in total energy per nucleon and in rigidity, and lies about midway between them. It is shown that using this form of source spectrum of cosmic ray nuclei, satisfactory agreement can be obtained between the calculated values and the observed ones of the ratios of H\textsuperscript{2}/He\textsuperscript{4} and He\textsuperscript{3}/He\textsuperscript{4}, and the energy spectra of protons and helium nuclei near the Earth.

1. Introduction

In all studies of the propagation of cosmic ray nuclei through interstellar and interplanetary space it is necessary to assume the form of the energy spectrum injected by cosmic ray sources, which is referred to as 'source spectrum'. The spectral form is assumed to be one of the following: (a) a power law in kinetic energy per nucleon, (b) a power law in magnetic rigidity and (c) a power law in total energy per nucleon. At relativistic energies all the three forms of the spectra are indistinguishable, but these differ significantly from one another at non-relativistic energies. The source spectrum of cosmic ray nuclei at low energies is not accessible to experimental observations due to the fact that processes, like ionization loss of cosmic ray nuclei in the interstellar medium and solar modulation in the interplanetary space, significantly affect the low energy particles and modify the energy spectrum. In the most recent reports of the analysis of the observational data of the energy dependence of the ratios of H\textsuperscript{2}/He\textsuperscript{4} and He\textsuperscript{3}/(He\textsuperscript{3} + He\textsuperscript{4}) and the energy spectra of protons and helium nuclei by Comstock et al. (1971), Ramadurai and Biswas (1971), and Meyer (1971), it is found that a source spectrum which is about midway between a power spectrum in total energy per nucleon and a power spectrum in rigidity can satisfactorily explain all these observational data. In this paper we show that a spectral form of this type can be derived on the basis of Fermi's theory itself.

2. Fermi Spectrum

It is well known that in the acceleration of cosmic ray particles by Fermi mechanism the total energy \( W \), after \( N \) collisions of a particle of rest energy \( Mc^2 \) is given (cf. Fermi, 1949) by

\[
W = Mc^2 \exp (B^2N),
\]  

(1)


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where $Bc$ is the velocity of the magnetic inhomogeneities. The number of collisions, $N$, after a time $t$ is equal to $t/\tau$ where $\tau$ is the mean time between successive collisions of the particle with the magnetic inhomogeneities. The mean collision time will depend on the distance between the magnetic inhomogeneities, $L$, and the velocity of the particle $\beta c$. Hence Equation (1) can be written as

$$W = M c^2 \exp \left( B^2 t / \tau \right) = M c^2 \exp \left( a \beta t \right),$$

where $a = B^2 c / L$ is called the acceleration parameter. It can be easily seen from Equation (2) that the rate of gain of energy is given by

$$\frac{d\gamma}{dt} = \frac{a \left( \gamma^2 - 1 \right)^{3/2}}{\left( \gamma^2 - 1 - \ln \gamma \right)},$$

where $\gamma = W / M c^2$.

In the case of relativistic particles the rate of energy gain reduces to

$$\frac{d\gamma}{dt} = a \gamma,$$

which is equivalent to the original expression derived by Fermi (1949). Since Fermi was considering only relativistic particles he did not include the velocity dependence of the acceleration parameter. However, at non-relativistic energies, this becomes very important as can be seen from the expression for the energy spectrum of cosmic ray nuclei derived below.

Let us assume that particles are injected into the accelerating region at a uniform rate over the mean lifetime $T$ of the particles in the accelerating region.

Let us assume that the particles are removed from the accelerating region according to an exponential law. Then the age distribution of particles at any time $t$ is given by

$$N(t) dt = N_0 \exp \left( - t / T \right) dt / T,$$

where $N_0$ is the number of particles injected above the critical energy.

The time dependence in expression (4) can be translated into the energy dependence by making use of expressions (2) and (3). Further, noting that $\ln \gamma$ will be always much smaller than $\gamma^2$, one can obtain after some algebraic transformations, the expression for the energy spectrum of particles injected after acceleration as given by

$$N(W) dW = K \left( M c^2 \right)^{1 + 1 / \alpha \beta T} \frac{1}{a \beta T} \frac{1}{W^{1 + 1 / \alpha \beta T}} dW.$$

It can be seen that the above expression reduces to the original expression derived by Fermi (1949) for relativistic particles with $\beta \approx 1$.

In order to use this form of spectrum we need to know the values of acceleration parameter $a$ and the mean life-time $T$. These quantities are not subject to direct obser-