POPULATIONS OF HIGHLY EXCITED ATOMS AND IONS IN THE OPTICALLY THIN PLASMA

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Abstract. Treating it as a boundary-value problem, the equations of highly excited state populations of atoms and ions are theoretically studied in case of recombining plasma. Scattering and spontaneous transitions as well as those induced by background radiation, are taken into account in the kinetic equations. The kinetic coefficients for inelastic scattering of incident charged particles on highly excited atoms and ions have been calculated in the asymptotically exact case: $1.6 \times 10^5 (z/n)^2 T_e^{-1} \ll 1$.

The distribution functions over the Rydberg states, analytically found, allow to determine amplification factor and optical depth of radio-recombination lines as functions of cosmic plasma parameters.

1. Introduction

In view of a great number of experimental reports on detection radio-recombination lines within wide wavelength range from centimeter up to decameter, theoretical analysis of line intensities as functions of cosmic-medium parameters and Rydberg states seems to be of special value. For optically thin plasma, line intensity depends strongly upon particle distribution over Rydberg levels which is found in analytic way in this paper.

Numerical solutions of kinetic equations for populations of highly excited states of atoms have been obtained for some models of plasma already (Sejnowsky and Hjellming, 1969; Shaver, 1975; Walmsley and Watson, 1982), so limiting other possible diagnostic applications. Additionally, some uncertainty in numerical solutions appears due to different approximations of collisional-transition rates, taken into account (Gee et al., 1976). In this work analytic calculation of the kinetic factors has been made with exactness, given by semi-classical description, which is about $n^{-1}$ for $n \gg 1$.

Analytically found, the kinetic factors have been compared also with numerical approximations (Gee et al., 1976; Sejnowski and Hjellming, 1969). The main physical cause for additional calculation of the kinetic factors lies in the fact that principal contribution to the excited particle settlement is made by such inelastic impacts when energy changing depends on the energy of excited state, so that $1 \ll \Delta n(n, T_e) \ll n$. In the limits of the 'collisional-spontaneous' model populations of highly excited levels have been theoretically investigated already (Vainstein et al., 1979; Beigman, 1987).

In the present paper, collisional, spontaneous, and induced by background radiation transitions are put into description as substantional ones in determining atomic and ionic populations in the interstellar medium whether in the cool phase $T_e = 20$–$100$ K or in the hot one $T_e = 10^6$–$10^8$ K. With all that, the balance equations as functions of highly excited state energy are resolved in analytic form with the help of some variant of the boundary method analogous to the Hilbert–Privalov’s problem.

The solutions found can be applied for interpretation of radio-recombination lines. They are of special interest for decameter and meter recombination lines, forming by transitions between super-high levels \( n = 500-700 \) in the works of Konovalenko and Sodin (1981), Konovalenko (1984), or Ershov et al. (1984).

The theoretical results are produced in the following way: in Section 2 the integral kinetic equation, averaged up on quantum states, is formulated in convolution form; in Section 3 diffusion factors and particle flows among excited states are calculated for interactions both with charged particles of plasma and galactic background radiation; in Section 4, split up in two parts; first all analytical solutions of kinetic equation are deduced at great length in order to get simple expressions for particle distribution and amplification factors of lines; secondly, results are plotted with other authors results on pictures to compare and discuss the physical situation; in Section 5, except for the principal conclusions, some estimates on amplification factors of radio-recombination lines of heavy elements have been carried out to provide for hot gas in supernova remnants.

2. Kinetic Equations for Rydberg Atoms

After a search for the radiative-collisional models of recombination in optically-thin plasma (Beigman, 1987), it is known that the ground-state decay of highly excited states is caused by spontaneous processes; by the continuum boundary inelastic collisions with small energy change as well as interaction with low-frequency, nonthermal radiation field; for example, the galactic background predominate over others. Provided that, by recombining plasma – when settlement of excited particles from the ground states is impossible – the principal source of Rydberg electrons from thermalized continuum should be a photo-recombination process.

Analytical approach to the balance equations is based on several assumptions, mostly made (e.g., Beigman, 1987):

1. Degenerate levels are uniformly populated over angular momenta \((l, m)\).
2. Distribution functions for Rydberg levels should be smooth in principal quantum number \( n \) or of energy of highly excitated states \( \epsilon \).
3. In the limit of \( n \to \infty \) \((\epsilon \to 0)\), the Fokker–Planck diffusion approximation should be adopted as correct for particle flow along some energy spectrum. Thus, with regard to the assumption (1), we can write the balance equations depending on the principal quantum number \( n \) in the form

\[
\sum_{n' = n+1}^{\infty} A_{nn'} \phi_{n'} - \sum_{n' = n_0}^{n-1} A_{nn'} \phi_n + \sum_{n' = n_0}^{\infty} [W_{nn'}^r + W_{nn'}^\epsilon] \phi_{n'} - \\
- \sum_{n' = n_0}^{\infty} [W_{n'n}^r + W_{n'n}^\epsilon] \phi_n = - F(n),
\]

where \( \phi_n \) describes the number of particles on the level \( n \); \( A_{nn'} \) is probability of spontaneous transition from level \( n \) to \( n' \); \( W_{nn'}^r \) and \( W_{nn'}^\epsilon \) are induced by radiation and