BREAK-UP OF QUASI-PERIODIC MOTION AS AN INTERPRETATION OF WHITE-DWARF VARIABILITY

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Abstract. High-speed photometric data obtained for a number of white-dwarf stars confirm the presence of subharmonic frequency in their power spectra. As many of these white dwarfs show multiperiodic structures, we show that the break up of quasi-periodic motion results in the appearance of similar subharmonic components in their spectra.

1. Introduction

Recent photometric observations of many white dwarfs show periodic, quasi-periodic, and irregular motion in their spectra (Sion, 1986; Winget, 1988). There has subsequently been a great deal of work to account for the observed variability of white dwarfs in terms of nonlinear dynamics (cf. Auvergne and Baglin, 1985; Tavakol and Das, 1986, hereafter referred to as RTDM; Goupil et al., 1988; Das et al., 1989; Vanclaire et al., 1989). These works mainly consider nonlinear oscillators or weak nonlinear couplings of such oscillators as models for the observed complex behaviour. The observation of more than two frequencies in the light curve of some white dwarfs (for example, G 185 – 32 shows up to five independent frequencies in its power spectrum (McGraw et al., 1981)) suggested the occurrence of quasi-periodic motion in dissipative regimes. It was subsequently shown that when the nonlinear coupling is small, quasi-periodic motion with up to five irrationally related frequencies can occur (cf. RTMD) in such stars, which can naturally be characterized as quasi-periodic motion on N-tori ($T^N$) with $N$ ranging from 1 to 5. It was also shown that as the strength of the coupling is increased, the quasi-periodic motion breaks up leading ultimately to chaotic motion.

Recently, Goupil et al. (1988) and Vauclaire et al. (1989) have found examples of white dwarfs (e.g., PG 1351 + 489 and G 191 – 16, respectively) whose spectra show, in addition to the fundamental frequency, also its subharmonics. They rigidly propose the relevance of nonlinear dynamics and, by considering a one-zone model for such stars, put forward the period doubling route to chaos as the appropriate mechanism.

Here we consider a more general setting where there are more than one oscillating modes being excited in such stars, by considering a set of five nonlinear oscillators with weak nonlinear couplings, and looking briefly at the break up of the resulting quasi-periodic motion. Our reasons for this are twofold: first, it is generically expected that...
more than one mode is activated in such stars. As a result the study of such break ups
in presence of other modes can be of relevance in the interpretation of the future
observations of such stars. Secondly, even in observations such as those of Goupil et al.
(1988) and Vauclaire et al. (1989) in which only one fundamental mode seems to be
observed, the presence of other frequencies with amplitudes below the noise level cannot
be ruled out. In either case it is important to show that this type of spectra can also exist
in more general regimes with their corresponding modes of break up.

2. Break-Up of Coupled Oscillators

The observed complex nonstationary spectra of the light curves of some of the white
dwarfs such as PG 1707 + 42, PG 2131 + 066, PG 1654 + 160, PG 1159 − 035,
PG 0122 + 200, G 191 − 16, and PG 1351 + 489 (Bond et al., 1984; Winget et al.,
1984, 1985; Bond and Grauer, 1987; Goupil et al., 1988; Vauclaire et al., 1989) suggest that
nonlinear effects might be of relevance for their explanation. In this connection, to
account for the luminosity variations as observed in the photometric data of white dwarf
PG 1351 + 489, Goupil et al. (1988), have put forward the period doubling scenario as
the mechanism for the production of such spectra and have motivated this by the
theoretical consideration of the modified Baker one-zone model for radial pulsations.

On the other hand it is known that motion with up to five independent modes of
oscillation have been formed in G 185 − 32. The quasi-periodic interpretation of this
type of motion was established earlier (cf. RTMD). To study the break up of such
quasi-periodic motion we consider, for simplicity, a polytropic model which models the
white-dwarf interiors fairly well for high- and low-central densities depending upon the
polytropic index $\nu$. In particular, we take a very centrally condensed white dwarf
approximated by a model of index $\nu = 3$, and consider nonlinear interaction of the first
five radial modes in the nonadiabatic regime. The nonadiabatic contributions to the
various modes have been modelled similar to that of Van der Pol oscillators (cf. RTDM;
Das et al., 1989; Takeuti, 1990). The basic oscillator equations for the truncated five
modes nonlinearly coupled oscillators could be written (cf. RTMD) as

$$
\dot{X}_1 = -X_1 + 0.9897\lambda_1 X_1^2 + 0.6547\lambda_2 X_1 X_2 + 0.0796\lambda_3 X_1 X_3 + \\
\mu_1(1 - X_1^2)\dot{X}_1 ,
$$

$$
\dot{X}_2 = -1.8354 X_2 + 4.738 \left(\frac{\lambda_1^2}{\lambda_2}\right) X_1^2 + 7.533\lambda_1 X_1 X_2 + \\
+ 2.435 \left(\frac{\lambda_1 \lambda_3}{\lambda_2}\right) X_1 X_3 + \mu_2(1 - X_2^2)\dot{X}_2 ,
$$

(1)

$$
\dot{X}_3 = -3.079 X_3 + 4.301 \left(\frac{\lambda_1^2}{\lambda_3}\right) X_1^2 + 16.855\lambda_1 X_1 X_3 + \\
+ 18.175 \left(\frac{\lambda_1 \lambda_2}{\lambda_3}\right) X_1 X_2 + \mu_3(1 - X_3^2)\dot{X}_3 ,
$$