THERMOSOLUTAL-CONVECTIVE INSTABILITY OF A COMPOSITE PLASMA IN A STELLAR ATMOSPHERE INCLUDING THE EFFECTS OF VARIABLE MAGNETIC FIELD AND ROTATION

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(Received 12 December, 1990)

Abstract. Thermosolutal-convective instability of a composite plasma in a stellar atmosphere is considered. The effect of a variable horizontal magnetic field and the simultaneous effect of a uniform rotation and a variable horizontal magnetic field have been considered on the thermosolutal-convective instability. We have derived the sufficient conditions for the existence of monotonic instability. It is found that the criteria for monotonic instability hold good in the presence of a variable horizontal magnetic field as well as in the presence of a uniform rotation and a variable horizontal magnetic field.

1. Introduction

The instability in which motions are derived by buoyancy forces of a thermally-unstable atmosphere has been termed as thermal-convective instability. Defouw (1970) generalized the Schwarzschild criterion for convection to include departures from adiabatic motion and has shown that thermally-unstable atmosphere is also convectively unstable, irrespective of the atmospheric temperature gradient. Defouw (1970) has found that a stellar atmosphere is monotonically unstable if

\[ D = \frac{1}{C_p} (L_T - \rho \alpha L_\rho) + \kappa k^2 < 0, \]

where \( L \) is the heat loss function and \( C_p, \rho, \alpha, \kappa, k, L_T, L_\rho \) denote, respectively, the specific heat at a constant pressure, the density, the coefficient of thermal expansion, the coefficient of thermometric conductivity, the wave number of the perturbation, the partial derivative of \( L \) with respect to temperature \( T \), and the partial derivative of \( L \) with respect to density \( \rho \), both evaluated in the equilibrium state. In general, the instability due to inequality (1) may either be oscillatory or monotonic. The effects of a uniform rotation and a uniform magnetic field on thermal-convective instability of a stellar atmosphere have been studied separately by Defouw (1970) and simultaneously by Bhatia (1971).

A detailed account of thermal convection, under varying assumptions of hydrodynamics and hydromagnetics, has been given by Chandrasekhar (1961). Veronis (1965) has treated the problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient. The thermohaline convection in a horizontal layer of viscous fluid heated from below and salted from above has been studied by Nield.
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(1967). Sharma and Misra (1986) have studied the thermosolutal-convective instability in a stellar atmosphere. In thermosolutal-convective instability problem, buoyancy forces can arise not only from density differences due to variations in temperature but also from those due to variations in solute concentration. The conditions under which convective motions are important in stellar atmospheres are usually far removed from the considerations of a single-component fluid and rigid boundaries and, therefore, it is desirable to consider the gas component acted on by solute-concentration gradient and free boundaries. In the above studies, a fully-ionized plasma has been considered. Quite frequently, the plasma is not fully ionized and may, instead, be permeated with neutral atoms. As a reasonably simple approximation, the plasma may be idealized as a mixture of a hydromagnetic (ionized) component and a neutral component, the two interacting through mutual collisional effects.

In the present paper we consider the thermosolutal-convective instability of a composite plasma in a stellar atmosphere. In stellar interiors and atmospheres, the magnetic field may be variable and may altogether alter the nature of the instability. The effect of a variable magnetic field has been considered on the thermosolutal-convective instability. The Coriolis force plays an important role in astrophysical problems. The combined effect of a variable magnetic field and a uniform rotation is also included.

2. Perturbation Equations

Consider an infinite horizontal viscous and finitely conducting composite fluid layer of density \( \rho \) permeated with neutrals of density \( \rho_d \), subjected to a stable solute concentration gradient and acted on by gravity force \( g(0, 0, -g) \) and a variable horizontal magnetic field \( \mathbf{H}(H_0(z), 0, 0) \). This layer is heated from above such that a steady temperature gradient \( \beta = \frac{dT}{dz} \) is maintained. The layer is soluted from below such that a steady solute concentration gradient \( \beta' = -\frac{dC}{dz} \) is maintained. Regarding the model under consideration we assume that both the ionized fluid and the neutral gas behave like continuum fluids and that effects on the neutral component resulting from the presence of gravity and pressure are neglected. The linearized perturbation equations governing the motion of the mixture of the hydromagnetic fluid and neutral gas are

\[
\frac{\partial \mathbf{q}}{\partial t} = -\frac{1}{\rho_0} \nabla \rho - g(\alpha \theta - \alpha' \gamma) + v \nabla^2 \mathbf{q} + \frac{\mu_e}{4\pi \rho_0} \left[ (\nabla \times \mathbf{h}) \times \mathbf{H} + (\nabla \times \mathbf{H}) \times \mathbf{h} \right] + \frac{\rho_d v_c}{\rho_0} (\mathbf{q}_d - \mathbf{q}) ,
\]

(2)

\[
\frac{\partial \mathbf{q}_d}{\partial t} = -v_c (\mathbf{q}_d - \mathbf{q}) ,
\]

(3)

\[ \nabla \cdot \mathbf{q} = 0 , \quad \nabla \cdot \mathbf{h} = 0 , \]

(4)

\[
\frac{\partial \gamma}{\partial t} - \kappa' \nabla^2 \gamma = \beta' \omega ,
\]

(5)