RADIATIVE HEAT TRANSFER TO HYDROMAGNETIC FLOW OF A SLIGHTLY RAREFIED BINARY GAS IN A VERTICAL CHANNEL

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Abstract. The paper considers the fully-developed slip flow in a vertical channel with radiative heat transfer and mass transfer in the presence of an externally applied magnetic field. The problem is modelled by the compressible Navier-Stokes equations, so that the gas is only slightly rarefied. Invoking the exact integral equation for radiation, the problem is reduced to a set of ordinary integro-differential equations. By realistic assumptions, the set is linearized and the temperature is reduced to a mixed Fredholm-Volterra integral equation which is solved by standard iterative procedure. Thereafter the concentration equation is solved by the WKB approximation while the velocity is obtained by the finite difference scheme. These solutions are discussed qualitatively.

1. Introduction

Frequently, and fairly approximately so, the study of the re-entry problem is modelled by constant density flow with radiative heat transfer. The incompressible Navier-Stokes equations then form the basis of the analysis (see Bestman, 1990a). However, in situations that prevail during re-entry, the density is quite low and the effect of gas rarefication may be considered. This paper is addressed to this problem which considers hydromagnetic flow of a slightly rarefied gas with radiative heat and mass transfer.

Thus in Section 2 the governing equations are presented in non-dimensional form and the solution methodology of these equations are presented. Section 3 is then devoted to the discussion of the results.

2. Governing Equations and their Solutions

Here we start by stating the non-dimensional equations of motion. Thus the configuration involves flow in a vertical channel. One wall of the channel is on \( y = 0 \) while the other wall is on \( y = 1 \). It is assumed that the flow is fully developed, the wall temperatures are constant at \( \theta_0 \) and the flow obeys the compressible Navier-Stokes equation with slip boundary conditions, hence, the gas is only slightly rarefied. Furthermore, there is mass transfer with chemical reaction and Arrhenius activation energy. If we denote the velocity by \( u \), pressure by \( p \), temperature by \( \theta \), concentration by \( c \), and radiative flux by \( q \), then following Bestman (1990b, c) the governing equations could be written in the
These equations assume that the induced magnetic field is negligible, which is so when the magnetic Reynolds number is small. The problem depends on the magnetic parameter $M^2$, the Froude number $F\infty$, the Mach number $M\infty$, the Prandtl number $Pr$, the Schmidt number $Sc$, the radiation parameters $B_0$ and $N$, the chemical reaction rate constant $k_r$ in the Arrhenius term where $\eta$ is a parameter and $E$ is the activation energy. Also $E_u(z)$ is the exponential integral, sgn$(z)$ is signum function, and $\gamma$ is the ratio of the specific heats.

The slip boundary conditions as given by Shidlovskiy (1967) are

$$u = \frac{2 - \chi}{\chi} \frac{5\pi}{16} K_n \frac{du}{dy} \quad \text{on} \quad y = 0, 1, \quad (5a)$$

$$\frac{\theta}{\theta_0} = 1 + \frac{2 - \lambda_\nu}{\lambda_\nu} \frac{75\pi}{128} K_n \frac{1}{\theta} \frac{d\theta}{dy} \quad \text{on} \quad y = 0, 1, \quad (5b)$$

$$\frac{c}{c_{0,1}} = 1 + \frac{2 - \lambda_c}{\lambda_c} \frac{75\pi}{128} K_n \frac{1}{\theta} \frac{dc}{dy} \quad \text{on} \quad y = 0, 1, \quad (5c)$$

in which $K_n$ is the Knudsen number, $\chi, \lambda_\nu,$ and $\lambda_c$ are the coefficients of slip for velocity, temperature, and concentration, respectively. Since the pressure is constant, the slip boundary condition for pressure is not applicable here. Also $c_{0,1}$ are the concentrations at the walls at $y = 0, 1$. We start the solution by noting that the last two relations in Equation (1) give

$$\rho = \frac{1}{\theta}, \quad p = \frac{1}{\gamma M^2 \infty}. \quad (6)$$

Hence, once $\theta$ has been determined, the solution for $u$ and $c$ can be obtained by finite difference scheme complimented by the WKB approximation. We develop the solution