NUMERICAL DETERMINATION OF ASYMMETRIC PERIODIC SOLUTIONS

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Abstract. A simple predictor-corrector procedure is described for the determination of 'asymmetric' periodic solutions of dynamical systems of two degrees of freedom. An application in the case of the Störmer problem is given. The computed periodic motions of the charged particle are of the open-path type.

1. Introduction

For certain dynamical systems of two degrees of freedom general solutions are impossible to obtain analytically and insight into the nature of such solutions can be gained only by means of numerical integration. The determination of periodic solutions and of their stability parameters is usually the main object (with reference to the restricted three-body problem see, e.g., Szebehely, 1967). This is due to well known reasons of feasibility and usefulness. In particular, symmetric periodic solutions have been determined in abundance because of the implied reduction of the number of free variables to be considered in the iteration schemes employed for the solution of the corresponding boundary value problems, and mainly because of the resulting economy in computer time. In the case of the Newtonian forces of the restricted three-body problem the symmetry is with respect to the synodical line of the primaries. A general discussion of symmetry (mirror) configurations and their relation to periodicity, can be found in Roy and Ovenden (1955). In the case of the Störmer problem the symmetry is with respect to the intersection of the equatorial plane with the meridian plane in which motion is reduced.

Relatively few results exist on asymmetric periodic solutions even in the most investigated problem of three bodies. Detection of such solutions in the vicinity of symmetric periodic ones has been discussed by Hénon (1965), while Message (1959) has actually given many asymmetric periodic orbits associated with exterior commensurabilities in the Sun-Jupiter case of the restricted problem.

We are presenting here a simple predictor-corrector scheme for the numerical determination of asymmetric periodic solutions (or simply APS) of a typical dynamical system of two degrees of freedom and of the autonomous type.
\[ \dot{x} = f(x), \]
\[ x = (x_1, x_2, x_3, x_4), \quad f = (f_1, f_2, f_3, f_4), \]
\[ (x_3 = \dot{x}_1, x_4 = \dot{x}_2) \] possessing a first integral of motion
\[ I(x) = C. \]

Examples of the type of dynamical systems under consideration are:

The plane restricted three-body problem,
\[ f_1 = x_3, \]
\[ f_2 = x_4, \]
\[ f_3 = 2x_4 + x_1 - \frac{(1 - \mu)(x_1 + \mu)}{r_1^3} - \frac{\mu(x_1 + \mu - 1)}{r_2^3}, \]
\[ f_4 = -2x_3 + x_2 - \frac{(1 - \mu)x_2}{r_1^3} - \frac{\mu x_2}{r_2^3}, \]
\[ r_1 = [(x_1 + \mu)^2 + x_2^2]^{1/2}, \]
\[ r_2 = [(x_1 + \mu - 1)^2 + x_2^2]^{1/2}, \]
with the integral
\[ C = x_1^2 + x_2^2 + \frac{2(1 - \mu)}{r_1} + \frac{2\mu}{r_2} - x_3^2 - x_4^2; \]

and the (reduced) Störmer problem,
\[ f_1 = x_3, \]
\[ f_2 = x_4, \]
\[ f_3 = \left( \frac{1}{x_1} - \frac{x_1}{r^3} \right) \left( \frac{1}{x_1^2} + \frac{1}{r^3} - \frac{3x_1^2}{r^5} \right), \]
\[ f_4 = \frac{3x_1 x_2}{r^5} \left( \frac{x_1}{r^3} - \frac{1}{x_1} \right), \]
\[ r = (x_1^2 + x_2^2)^{1/2}, \]
with integral
\[ C = \frac{1}{2}(x_3^2 + x_4^2) + \frac{1}{2} \left( \frac{1}{x_1} - \frac{x_1}{r^3} \right)^2. \]

2. Asymmetric Periodic Solutions and their Stability

Such solutions can be represented by the fixed points of the mapping
\[ x_1 = F(x_{01}, x_{03}, x_{04}), \]
\[ x_3 = G(x_{01}, x_{03}, x_{04}), \]
\[ x_4 = H(x_{01}, x_{03}, x_{04}), \]
which are characterized by a non-zero value of \( x_{03} = x_3 \). The mapping is defined as the