TEST COMPUTATIONS ON THE DYNAMICAL EVOLUTION 
OF STAR CLUSTERS

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Abstract. Test calculations have been carried out on the evolution of star clusters using the fluid-dynamical method devised by Larson (1970a). Large systems of stars have been considered with specific concern with globular clusters. With reference to the analogous ‘standard’ model by Larson, the influence of varying in turn the various free parameters (cluster mass, star mass, tidal radius, mass concentration of the initial model) has been studied for the results. Furthermore, the partial release of some simplifying assumptions with regard to the relaxation time and distribution of the ‘target’ stars has been considered. The change of the structural properties is discussed, and the variation of the evolutionary time scale is outlined. An indicative agreement of the results obtained here with structural properties of globular clusters as deduced from previous theoretical models is pointed out.

1. Introduction

The dynamical evolutionary changes of large star clusters for which the N-body integration becomes inapplicable, can be treated by means of either the Monte Carlo method, or the fluid-dynamical approach. The first procedure allowed the calculation of a certain number of evolutions of various star clusters by some authors (e.g., Hénon 1966, 1967, 1971a, b; Spitzer and Hart, 1971a, b); the second approach was only applied to a limited number of models by Larson (1970a, b).

This latter method consists in a fluid-dynamical approach through the numerical integration of the moment equations derived from the Boltzmann equation (Larson, 1970a). The stellar system is assumed to possess spherical symmetry and the moments of the velocity distribution are extended up to the fourth order; ‘collision terms’ are taken into account by means of the Fokker–Planck equation. The method allows for the relaxation of a stellar system with increasing condensation in the central region and growing velocity anisotropy in the outer zone, as a consequence of the star encounters.

For a ‘standard’ stellar system mimicking a globular cluster with initial tidal radius $R = 100$ pc, total mass $M = 2 \times 10^5 M_\odot$ and consisting of one ‘population’ of stars with mass $m = 1 M_\odot$ each, Larson (1970b) computed the evolution by allowing for an escape of stars from the system by means of the consideration of a perfectly absorbing boundary.

The fluid-dynamical method appears to be fast (more than the Monte Carlo method) and capable of handling various situations; also it has the possibility to be extended to two (or more) populations of stars, as star masses are concerned. On the other hand, it has been used only by the cited author, who, with regard to globular
clusters, concentrated on changes of results as due to various initial density distributions. Therefore, we decided to increase the number of cases studied by varying some free parameters and releasing some simplifications made. The purpose was two-fold, (i) to test our independent computer program under various conditions, and (ii) to study the importance of changes of initial conditions and the influence of some simplifying assumptions for the results. It was also intended that some new cases can help in the comparison between the fluid-dynamical method and other methods (Aarseth et al., 1974). Furthermore, with the intention to generalize the method to clusters with more populations of star masses, various different cases are needed in order to make it possible to evaluate the dominant parameters.

Studies of this kind, even with the severe limitations set up by some of the assumptions, can provide useful information about the structure, evolutionary lifetime and rate of star escape from large star clusters.

2. Method and Results

We systematically refer to the notation of Larson (1970a), if not stated otherwise; the assumptions and approximations listed there are also maintained. In particular, it is usually assumed that the ‘target’ stars have an isotropic velocity distribution. In the paper quoted above the moment equations for $q$ (number density of stars), $\langle u \rangle$ (mean radial velocity), $\alpha \equiv \langle (u - \langle u \rangle)^2 \rangle$ and $\beta \equiv \langle v^2 \rangle = \langle w^2 \rangle$ (mean-squared random velocities in the radial and transverse directions), $\epsilon \equiv \langle (u - \langle u \rangle)^3 \rangle$ and $\xi = \zeta - 3\alpha^2$, with $\zeta \equiv \langle (u - \langle u \rangle)^4 \rangle$, were derived with regard to time $t$ and radial distance $r$. $\epsilon$ represents a radial energy transport (outward for $\epsilon > 0$, inward for $\epsilon < 0$), $\xi > 0$ an excess ($\xi < 0$ a deficiency) of high velocity stars relative to a Maxwellian distribution. In the following the mean radial velocity will be denoted by means of $\bar{u}$. The equations are

\begin{align*}
\frac{\partial q}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} r^2 q \bar{u} &= 0, \\
\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial r} + \frac{1}{q} \frac{\partial}{\partial r} q \alpha + \frac{2}{r} (\alpha - \beta) + \frac{\partial \Phi}{\partial r} &= 0, \\
\frac{\partial \alpha}{\partial t} + \bar{u} \frac{\partial \alpha}{\partial r} + 2\alpha \frac{\partial \bar{u}}{\partial r} + \frac{1}{q} \frac{\partial}{\partial r} q \epsilon + \frac{2 \epsilon}{r} \left(1 - \frac{2 \beta}{3 \alpha}\right) &= - \frac{4 (\alpha - \beta)}{T}, \\
\frac{\partial \beta}{\partial t} + \bar{u} \frac{\partial \beta}{\partial r} + 2\beta \frac{\bar{u}}{r} + \frac{1}{3q} \frac{\partial}{\partial r} q \epsilon + \frac{4 \epsilon}{3 \alpha} \frac{4 \beta}{3 \alpha} &= \frac{2 (\alpha - \beta)}{T}, \\
\frac{\partial \epsilon}{\partial t} + \bar{u} \frac{\partial \epsilon}{\partial r} + 3\epsilon \frac{\partial \bar{u}}{\partial r} + 3\alpha \frac{\partial \alpha}{\partial r} + \frac{1}{q} \frac{\partial}{\partial r} q \xi + \frac{2 \xi}{r} \left(1 - \frac{\beta}{\alpha}\right) &= - \frac{87 \epsilon}{160 T}, \\
\frac{\partial \xi}{\partial t} + \bar{u} \frac{\partial \xi}{\partial r} + 4\xi \frac{\partial \bar{u}}{\partial r} + 6\epsilon \frac{\partial \alpha}{\partial r} + 4\alpha \frac{\partial \epsilon}{\partial r} + 4\alpha \frac{\partial \xi}{\partial r} &= - \frac{3}{35} \left[7\xi - 15\alpha(\alpha - \beta)\right].
\end{align*}

(1)