A COMPARISON OF NUMERICAL INTEGRATION METHODS
IN THE EQUATORIAL MAGNETIC-BINARY PROBLEM

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Abstract. In this paper we compare several Runge–Kutta type methods of fourth-order when they are applied in the Equatorial Magnetic-Binary Problem (EMBP) for which the analytical solutions are not known. The results of the comparison are presented with tables and diagrams, and the most efficient method is proposed according to the used criteria which are described.

1. Introduction

It is well known that a great number of numerical methods, for the approximate solution of a first-order system of differential equations

\[ \dot{y} = f(t, y), \quad y(t_0) = y_0, \]  

have been proposed mainly the last thirty years. Therefore, there is much confusion in the literature about which is the best algorithm for handling a given differential system. This fact obligated in the past several researchers to compare various numerical methods on the base of different criteria and for particular classes of ordinary differential equations. See, for example, Shampine and Watts (1971), Hull et al. (1972), Jackson et al. (1978), Papageorgiou (1983), Fox (1984), and others. Also the appearance of new methods and the study of new problems in the various areas of the science, leave the problem of comparison of the various methods almost open. Besides, we believe that the worth of a numerical method is shown forth mainly, under the successful application of it for the solution of non-trivial problems.

Therefore, we decided to apply a number of numerical methods of Runge–Kutta-type to approximate the solution of the Magnetic-Binary Problem which is relatively new and enough complicated, in order to distinguish the most efficient from them. In this spirit, this paper is intended to be a part of a research project, intending to contribute to the whole effort, and providing new knowledge to the investigators of the above problem.

The Magnetic-Binary Problem was initially stated by Mavraganis (1978), and deals with the motion of a charged particle in the field of two magnetic bodies disposing dipolar fields. The bodies (hereafter called the primaries) are revolving around their center of mass in circular orbits under their mutual Newtonian attraction. The complexity of the general three-dimensional problem, led Kalvouridis (1987) to consider at first the planar motion of the particle which takes place on the equatorial planes of the...
primaries. The equations which describe this particular motion are given in the next section.

The methods considered in this paper are the explicit Runge-Kutta-type methods of order four. A subroutine library usually has at least one program based on such a formula. In the past, the classical fourth-order formula of Kutta (1901), or a variant of Kutta's method due to Gill (1951) were used. The estimation of the error for these methods is based on the technique of Richardson (1927) which consists on the comparison of two approximations from two half steps and one full step. More recently, formula pairs have been gaining in popularity. They consist of two formulas which use the same set of function evaluations and enable the user to obtain both an approximation to the solution of the differential system and an estimate of the local error, without having to re-apply the method with a smaller stepsize. The error estimate is used for the automatic change of the step. This idea seems to have been proposed originally by Merson (1957). Subsequently, formula pairs of various orders have been proposed by Scraton (1964), Zonneveld (1964), Shintani (1966), Fehlberg (1968, 1969a, b, 1986), England (1969), Bettis (1978), Verner (1978, 1979), Dormand and Prince (1980), and others.

For our comparison we have selected and applied six of the well-known Runge-Kutta formulas which have been proposed by Merson (1957), England (1969), Zonneveld (1964) Fehlberg 1 (1969b), Fehlberg 2 (1969b), and Dormand and Prince (1980) and all are of the fourth order.

2. Equations of Motion of the Equatorial Magnetic-Binary Problem

To describe the equatorial motion of a particle in a Magnetic-Binary system, it is convenient to consider in place of the synodic coordinate system $Oxyz$ a Cartesian frame of reference $P_i x_i y_i z_i$, $i = 1, 2$ which is fixed in the corresponding primary $P_i$ and oriented so as the $P_i z_i$-axis coincides with the dipole axis.

By means of the Lagrangian $L^i = T^i - V^i$, $i = 1, 2$ of the system, the equations which govern the particle motion can be written as

$$\frac{d}{dt} \left( \frac{\partial L^i}{\partial x_i} \right) - \frac{\partial L^i}{\partial x_i} = 0, \quad i = 1, 2,$$

where $T^i = \frac{1}{2}(\dot{x}_i^2 + \dot{y}_i^2)$, is the kinetic energy of the particle and

$$V^i = - b_1^i \dot{x}_i - b_2^i \dot{y}_i + V_0',$$

is the velocity-dependent potential function (Kalvouridis and Mavraganis, 1986).