Spherically-Symmetric Charged Perfect Fluid Distribution in Brans-Dicke Theory

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(Received 14 December, 1988)

Abstract. Solutions of Brans-Dicke field equations in Dicke's conformally-transformed units are obtained when the source of the gravitational field is that of charged perfect fluid where the line element taken is that of Robertson-Walker.

1. Introduction

The Brans-Dicke (BD) theory (cf. Brans and Dicke, 1961) was originally expressed in a form in which the equation of motion for test particles is identical to that of General Relativity and in this theory the gravitational constant $G$ is replaced with the reciprocal of a scalar field $\phi$. The latter is in turn determined by the trace of the energy-momentum tensor of all other non-gravitational fields (cf. Singh et al., 1983).

This theory (BD) can also be represented in a different form (cf. Dicke, 1962) through a unit transformation (UT) in which length, time, and reciprocal mass are scaled by a factor $\lambda^{-1/2}$ where $\lambda = \phi/\bar{\phi}$, $\bar{\phi}$ is a constant, so that $g_{\mu\nu} \rightarrow \lambda g_{\mu\nu}$, etc., one can obtain the field equations (cf. Raychaudhury, 1979) as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi(T_{\mu\nu} + \Lambda_{\mu\nu}),$$

$$\Lambda_{\mu\nu} = \frac{2\omega + 3}{16\pi\phi^2} \left[ \phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g_{\mu\nu}\phi_{,m}\phi^{,m} \right],$$

$$\Box (\ln \phi) \equiv (\ln \phi)_{,m} = \frac{8\pi T}{2\omega + 3},$$

where $\omega$ is the coupling constant and $T_{\mu\nu}$ is the usual energy-momentum tensor. Semicolons and commas followed by a suffix denote covariant and partial derivative, respectively.

In Section 2 we present the solutions of the field equations for a charged perfect fluid, and in Section 3 the physical interpretation of the solutions are given. We obtain a class of exact solutions for the case where the charge density $\varepsilon \neq 0$ and conductivity $\sigma \neq 0$.

2. Field Equations and their Solutions

To solve the field equations we assume a space-time with a spherically-symmetric distribution with maximally symmetric three-dimensional subspaces whose matrices

have positive eigenvalues and arbitrary curvature. This leads to the metric of the form (cf. Singh and Shridhar Deo, 1986)

\[ ds^2 = dt^2 - R^2(t) \left\{ \frac{dr^2}{1 - kr^2} + r^2 \, d\theta^2 + r^2 \sin^2 \theta \, d\phi^2 \right\}, \]  

(4)

where \( k \) is the curvature index which can take values \(-1, 0, \text{or} 1\) for the closed, flat, or open universe, respectively. The energy-momentum tensor \( T_{\mu\nu} \) is given by

\[ T_{\mu\nu} = M_{\mu\nu} + E_{\mu\nu}, \]

where

\[ M_{\mu\nu} = (p + \rho)u_\mu u_\nu - p g_{\mu\nu} \]  

(5a)

and

\[ E_{\mu\nu} = \frac{1}{4\pi} \left\{ -F_i^\alpha F_{j\alpha} + \frac{1}{4} g_{\mu\nu} F^{\alpha\beta} F_{\alpha\beta} \right\}; \]  

(5b)

where \( p \) is the isotropic pressure, \( \rho \) the fluid density, \( u_i \) the four-velocity vector of flow, and \( F_{ij} \) the electromagnetic field tensors. We also have

\[ u_i u^i = 1. \]  

(6)

Now considering the comoving coordinate system we have

\[ u^1 = u^2 = u^3 = 0 \quad \text{and} \quad u^4 = 1. \]  

(7)

The electromagnetic field equations are given by

\[ F_{i\alpha} = -J^i \]  

(8)

and

\[ F_{[\mu, \nu]} = 0, \]  

(9)

where \( J^i \) is the current four-vector and in general, is the sum of the convection current and conduction current (Tupper, 1977) – i.e.,

\[ J^i = \epsilon u^i - \sigma u^i F_i^\alpha, \]  

(10)

where \( \epsilon \) is the rest charge density and \( \sigma \) is the conductivity. As for notation, overhead ‘dot’ and ‘dash’, respectively, represents for derivatives with respect to time \( t \) and distance \( r \).

Now for the metric (4) the surviving field equations are

\[ 2 \frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = -8\pi p - 8\pi E_4^1 - 2\omega + \frac{3}{4} \left[ \frac{1 - kr^2}{R^2} \left( \frac{\phi'}{\phi} \right)^2 + \left( \frac{\phi'}{\phi} \right)^2 \right], \]  

(11)