SOLUTION OF ONE-FLUID MODEL EQUATIONS
WITH SHORT RANGE RETARDING MAGNETIC
FORCES FOR THE QUIET SOLAR WIND

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Abstract. The discrepancy of the low predicted versus the observed coronal particle densities is investigated by considering radial magnetic forces acting at the base of the corona in the one fluid model equations with anomalous thermal conductivity for the quiet solar wind. If the short range retarding magnetic force is taken to fall as \( r^{-5} \), \( r \) being the heliocentric distance, then in order to obtain satisfactory agreement between the predicted and observed (about \( 3 \times 10^8 \) cm\(^{-3} \) at \( R_o \)) coronal densities, the strength of the retarding magnetic force at \( R_o \) should be 1.2 times that of the gravitational force.

The 'conventional' one-fluid and the more adequate two-fluid model equations with spherically symmetrical flow were successful in predicting gross features of the quiet solar wind (Parker, 1958; Noble and Scarf, 1963; Whang and Chang, 1965; Sturrock and Hartle, 1966).

When 'non-conventional' one and two-fluid model equations were used, that is when anomalous transport coefficients allowing for contributions from non-collisional (i.e. wave-particle) interaction were incorporated into the model equations, a satisfactory quantitative agreement between theoretical predictions and observations between a distance several solar radii away from the Sun and 1 AU was obtained. This agreement refers to quantities such as particle density, streaming velocity, electron and proton temperature and electron heat flux (Cuperman and Harten, 1970, 1971a, 1971b; Cuperman et al., 1972; Hartle and Barnes, 1970; Wolff et al., 1971).

However, in all the above mentioned models coronal densities smaller by a large factor than those observed were predicted (conversely, when an agreement between theory and observations at the base of the corona (\( R_o \)) was obtained, the predicted densities were too large and the streaming velocities too low at 1 AU). This discrepancy between theoretical predictions and observations at the base of the corona has already been emphasized by several authors in the past (see for instance, Hundhausen, 1968; Parker, 1969, 1971). At \( R_o \) the discrepancy ranges between a factor of about 2.5 and about 30.

Inclusion of magnetic fields which are purely radial close to the Sun (Weber and Davis, 1967; Grzedzielski, 1969; Wolff et al., 1971) or of viscosity (Scarf and Noble, 1965; Whang et al., 1966; Wolff et al., 1971) in the model equations did not resolve the discrepancy between the predicted and observed coronal density. Thus, one may conclude that the discrepancy between the predicted (too low) and the observed coronal densities is a common feature of all the theoretical models for the quiet solar wind suggested so far.
In view of the satisfactory agreement between the predictions of non-collisional models and observations (except for a distance extending a few solar radii away from the Sun) we suggest that the one-fluid and two-fluid model equations with anomalous transport coefficients represent a satisfactory description for the quiet solar wind in the range starting a few solar radii away from the Sun's surface. At the base of the corona, short range retarding magnetic forces should be taken into account because the loop-like (closed) magnetic configurations existing there produce a retardation of the plasma particles which results in increased coronal densities. Because of their relatively short range, the magnetic forces should not affect the solutions in the main part of the interplanetary region, in which agreement with observations has already been achieved. This is consistent with the estimates by Schatten et al. (1968) which show the predominance close to the Sun of the transverse (and, therefore, also total) magnetic field energy density over the thermal and streaming energy and suggest the doubtful validity (close to the Sun) of theoretical models (and of their solutions) which neglect the existence of retarding magnetic forces.

Thus, in this work we consider retarding magnetic forces acting at the base of the corona in the one fluid model equations with anomalous thermal conductivity for the quiet solar wind.

The continuity, momentum and energy equations are numerically solved, subject to the assumption of steady symmetrical flow and omitting effects such as viscosity, solar rotation and fluctuations. These equations may be written as

\[ nvr^2 = \text{const.}, \]  
\[ nm \frac{dv}{dr} + \frac{d}{dr}(nkT) + \frac{GM_{\odot} n_m}{r^2} - \frac{1}{2} (j \times B)_r = 0, \]  
\[ \frac{3}{2} n v k \frac{dT}{dr} - nkT \frac{dn}{dr} - \frac{1}{2} \cdot \frac{d}{dr} \left[ r^2 K_{\text{mod}} \frac{dT}{dr} \right] = 0, \]

where the standard notation is used. \( K_{\text{mod}} \) represents a 'modified' (anomalously low) thermal conductivity allowed to include, besides the pure collisional contribution, (Spitzer, 1962) contributions due to wave-particle interactions and effects due to inhibition of transport across magnetic field lines (see Cuperman and Metzler, 1973).

The force term \(-\frac{1}{2} (j \times B)_r\) appearing in the momentum Equation (2) represents the short range retarding magnetic force acting at the base of the corona, as discussed above. In fact, it describes the equivalent effect of a rather complex coronal magnetic configuration based on loop-like, or other closed shape, substructures which may also change in time. For tractability of the problem, we choose to represent the 'equivalent' retarding magnetic force in a rather simple way. In the following, we take

\[ F_B \equiv -\frac{1}{2} \frac{1}{n} (j \times B)_r = (F_B)_{1 R_{\odot}} (R_{\odot}/r)^a \]

and express \((F_B)_{1 R_{\odot}}\) in terms of the gravitational force at the same position...