THE INSTABILITY OF A HORIZONTAL MAGNETIC FIELD IN AN ATMOSPHERE STABLE AGAINST CONVECTION*

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(Received 16 November, 1978)

Abstract. The theoretical problem posed by the buoyant escape of a magnetic field from the interior of a stably stratified body bears directly on the question of the present existence of primordial magnetic fields in stars. This paper treats the onset of the Rayleigh–Taylor instability of the upper boundary of a uniform horizontal magnetic field in a stably stratified atmosphere. The calculations are carried out in the Boussinesq approximation and show the rapid growth of the initial infinitesimal perturbation of the boundary. This result is in contrast to the extremely slow buoyant rise of a separate flux tube in the same atmosphere. Thus for instance, at a depth of $1/3 \, R_\odot$ beneath the surface of the Sun, a field of $10^2$ G develops ripples over a scale of $10^9$ km in a characteristic time of 50 years, whereas the characteristic rise time of the same field in separate flux tubes with the same dimensions is $10^{10}$ years. Thus, the development of irregularities proceeds quickly, soon slowing, however, to a very slow pace when the amplitude of the irregularities becomes significant. Altogether the calculations show the complexity of the question of the existence of remnant primordial magnetic fields in stellar interiors.

1. The Problem

Earlier papers have noted the rapid buoyant rise of a flux tube in an atmosphere with neutral convective stability (Parker, 1975) and the slow buoyant rise in an atmosphere that is stable against convection (Parker, 1974). In the stable region beneath the convective zone of the Sun, for instance, a flux tube of $10^2$ G and a radius of $10^3$ km rises at a rate of the general order of $10^{-9}$ cm s$^{-1}$, whereas in the convective zone itself the same tube would rise at a rate of the order of 1 cm s$^{-1}$ or more. The buoyant escape of magnetic fields places severe limitations on the generation of fields in stars, and on the retention of a remnant of a primordial field in the stable core inside the convective zone (Parker, 1974, 1975, 1979). The purpose of the present paper is to pursue the problem further with a study of the instability of a horizontal magnetic field submerged in a stably stratified atmosphere. In particular, we are interested in the linear onset of the breakup of the upper surface of the field, eventually forming isolated flux tubes whose individual rate of rise has already been treated elsewhere (Parker, 1974).

The calculation is carried out in the Boussinesq approximation for an ideal fluid with infinite electrical conductivity and finite thermometric conductivity $\kappa$. The most

* This work was supported in part by the National Aeronautics and Space Administration under Grant NGL 14-001-001.
unstable modes are those which do not distort the lines of force of the magnetic field. If we choose the uniform field $B$ to lie in the horizontal $z$-direction, then the fluid motions are in the $x$, and $y$ directions and $\partial \delta \vec{z} / \partial z = 0$ for the fastest modes. Point the $y$-axis upward so that the gravitational acceleration is $-\vec{e}_y g$. Place the $xz$ plane at the upper boundary of the magnetic field, so that the fluid is field-free and of uniform density $\rho$ in $y > 0$. For $y < 0$ there is the uniform magnetic pressure $B^2/8\pi$ which reduces the density by the small amount $\Delta \rho$. The small density reduction $\Delta \rho$ drives the Rayleigh–Taylor instability of the upper surface of the field in opposition to the stable temperature stratification

$$ T(y) = T_0(1 + y/L), \quad (1) $$

with gradient $T_0/L$. In the spirit of the Boussinesq approximation, the small density fluctuations are included only in the buoyancy term and not in the inertial terms.

The linearized equations for the fluid velocity $v$ and the heat flow are

$$ \rho \frac{\partial \delta v}{\partial t} = -\nabla \delta p + \rho g \delta \rho \frac{\partial T}{\partial y}, \quad (2) $$

$$ (\frac{\partial}{\partial t} - \kappa \nabla^2) \delta T = -(T_0/L)v_y, \quad (3) $$

where $\delta T$ is the temperature perturbation caused by the fluid motion $v$ in the ambient temperature gradient $\delta \rho / \rho$. These equations apply in both $y > 0$ and $y < 0$.

It is convenient to express the fluid velocity $v$ in terms of the stream function $\psi$ as

$$ v_x = + \frac{\partial \psi}{\partial y}, \quad v_y = - \frac{\partial \psi}{\partial x}. \quad (4) $$

Then the curl of (2) becomes

$$ \nabla^2 \frac{\partial \psi}{\partial t} = -\rho g \frac{\partial \delta T}{\partial x}. \quad (5) $$

Operating on this with $\partial / \partial t - \kappa \nabla^2$ and using (3) to eliminate $\delta T$ yields

$$ \frac{\partial}{\partial t} \left( \frac{\partial}{\partial t} - \kappa \nabla^2 \right) \nabla^2 \psi + \frac{\rho g T_0}{L} \frac{\partial^2 \psi}{\partial x^2} = 0. \quad (6) $$

Let

$$ \psi = \exp \left( t/\tau + i k x + i q y \right); \quad (7) $$

then (6) leads to the dispersion relation

$$ \frac{1}{\tau} \left[ \frac{1}{\tau} + \kappa k^2 (q^2 + 1) \right] (q^2 + 1) + \frac{\rho g T_0}{L} = 0. $$

It is convenient to work with the dimensionless growth rate $\Omega = 1/k V_A \tau$, the dimensionless conductivity $K = \kappa k / V_A$, and the dimensionless stability parameter $M = \rho g T_0 / L k^2 V_A^2$, where $V_A$ is the characteristic Alfvén speed $B/(4\pi \rho)^{1/2}$; if so, the dispersion relation can be written as

$$ \Omega [\Omega + K(q^2 + 1)](q^2 + 1) + M = 0. \quad (8) $$