GENERAL RELATIVITY AND FIFTH FORCE EXPERIMENTS

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Abstract. We calculate some general relativistic corrections to the Newtonian weak field limit of Einstein gravity, and discuss their possible relevance to considerations regarding the current set of fifth force experiments.

At the present time there is a great deal of interest in the possible existence of a very weak fifth fundamental force (Ander, 1989; Aronson, 1989; Hughes, 1989), with the experimental situation unfortunately not being very clear or conclusive. Now in all considerations of whether there may be new physics which is even weaker in strength than gravity, it is necessary to take into account higher order general relativistic effects, not only to see whether or not they may in fact be responsible for any experimental observations, but also to include their effects explicitly so that any residual otherwise unaccounted for effects could then be attributed to a possible fifth force. In this paper we shall calculate some general relativistic corrections to the Newtonian potential, and as we shall see, the obtained corrections will have a structure which could turn out to be relevant to fifth core considerations.

In general relativity the most general spherically-symmetric, static line element may be written in the form

$$\text{d}^2 c^2 = B(r) \text{ d}r^2 - A(r) \text{ d}r^2 - r^2 \text{ d}\Omega,$$

with the geodesic equation

$$\frac{d^2 x^\mu}{d \tau^2} + \Gamma^\mu_{\nu\lambda} \frac{dx^\nu}{d \tau} \frac{dx^\lambda}{d \tau} = 0,$$

reducing to

$$\frac{d^2 r}{d t^2} = -\Gamma^r_r = -\frac{1}{2A} \frac{dB}{d r},$$

for the radial acceleration in a static configuration. For the familiar Schwarzschild metric

$$B(r) = \frac{1}{A(r)} = 1 - \frac{2MG}{r},$$

Equation (3) yields
\[ \frac{d^2 r}{dt^2} = - \frac{d}{dr} \left( \frac{MG}{r} + \frac{M^2 G^2}{r^2} \right), \] (5)
and thus we can think of the quantity \( V_S(r) \) given by
\[ V_S(r) = - \frac{MG}{r} + \frac{M^2 G^2}{r^2}, \] (6)
as serving as the appropriate potential for the Schwarzschild geometry. As we can see, \( V_S(r) \) contains a general relativistic correction to the Newton term. At the surface of the Earth this correction is a factor \( 7 \times 10^{-10} \) smaller than the Newtonian potential of the Earth itself at its surface. Hence, general relativity naturally leads to small corrections to the Newtonian weak field limit*.

Having seen that in principle there are general relativistic corrections we can ask whether it is possible to get even larger ones. In a recent paper Mannheim and Kazanas (1990) noted that the Schwarzschild vacuum solution will actually be modified in non-leading order if the vacuum is not simply empty and inert but instead is a collective Higgs medium. Specifically, they considered the extremely simple model of a scalar field \( S(x) \) coupled to gravity via the action
\[ I = - \int d^4 x (-g)^{1/2} \left( \frac{\kappa}{2} R^z + \frac{1}{2} S_{\alpha} S^\alpha \right), \] (7)
where \( \kappa \) is an appropriate dimensional parameter, and obtained a self-consistent solution in which both the geometry and the scalar field depended on the radial coordinate \( \rho \). In convenient isotropic coordinates where the metric takes the form
\[ d\tau^2 = H(\rho) \, dt^2 - J(\rho) \, (d\rho^2 + \rho^2 \, d\Omega), \] (8)
their solution is given by**
\[ S(\rho) = \frac{C}{2r_0} \ln \left( \frac{\rho - r_0}{\rho + r_0} \right) + S_0, \] (9)

* There are other corrections which we do not consider here. First, the density and shape of the Earth are not in fact exactly spherically-symmetric and, hence, we cannot quite treat the total gravitational potential of the Earth as though it were due to single mass located at its center. (This problem, as well as the problem of actually knowing the gravitational potential at the surface of the Earth, can partially avoided by doing Pound–Rebka-type experiments at the same location where any fifth force experiment is being performed. Then the general relativistic frequency shift could be used to measure the local gravity and its local gradient in the first place.) Secondly, the Earth carries angular momentum \( J \) and, hence, the correct general relativistic vacuum metric to consider is the Kerr metric rather than the Schwarzschild one. This leads to a correction to \( g_{00} \) of the form \( g_{00} = -1 + (2MG/r)(1 + \cos^2 \theta J^2/M^2 r^2) \). For the Earth the factor \( J^2/M^2 r^2 \) is of order \( 10^{-10} \) at its surface.

** We set the irrelevant parameter \( a \) given in the solution presented by Mannheim and Kazanas (1990) equal to unity here.