FOURIER ANALYSIS OF THE LIGHT CURVES OF ECLIPSING VARIABLES, XII

ZDENĚK KOPAL
Dept. of Astronomy, University of Manchester, England*

(Received 20 April, 1977)

Abstract. The aim of the present paper will be to make use of the expressions, established in Paper XI, for the fractional loss of light \( \alpha_0 \) of arbitrarily limb-darkened stars in the form of Hankel transforms of zero order, in order to evaluate the explicit forms of the \( \alpha_0 \)'s for different types of eclipses (Section 2), as well as of the moments \( A_{2m} \) of the respective light curves (Section 3) – in a closed form; or in terms of expansions that converge under all circumstances envisaged. Particular attention will be directed to a connection between these expansions and other functions already available in tabular form; or to alternative forms amenable to automatic computation.

1. Introduction

In a preceding paper of this series (Kopal, 1977) we have shown that the fractional loss of light \( \alpha_0 \) during eclipse of an arbitrarily darkened stellar disc can be expressed (cf. Equation (2.32) of Paper XI) as a Hankel transform of the form

\[
\alpha_0^*(r_1, r_2, \delta) = 2^* \Gamma(v) \int_0^\infty \left(2\pi qr_1\right)^{-v} J_v(2\pi qr_1) \times
\]
\[
\times J_1(2\pi qr_2) J_0(2\pi q\delta) d(2\pi qr_2),
\]

(1.1)

where

\[
v \equiv \frac{l + 2}{2}
\]

(1.2)

denotes a constant related with the degree \( l \) of the law of limb darkening; \( r_1, r_2 \) stand for the radii of the eclipsed and eclipsing component, respectively; and \( \delta \), for the separation of their projected centres. If, moreover, we set

\[
2\pi qr_2 \equiv x,
\]

(1.3)

the foregoing transform (1.1) can be rewritten as

\[
\alpha_0^* = 2^* \Gamma(v) \int_0^\infty (kx)^{-v} J_v(kx)J_1(x)J_0(hx) \, dx,
\]

(1.4)

* Guest Investigator, U.S. Naval Research Laboratory, Washington, D.C.
where we have abbreviated

\[ h \equiv \frac{8}{r_2} \quad \text{and} \quad k \equiv \frac{r_1}{r_2}. \]  

(1.5)

The aim of the present paper will be to evaluate the function as defined by Equations (1.1) or (1.4) in terms of the parameters \( h \) and \( k \) (or their equivalents) for any type of eclipse and any degree of limb-darkening of the eclipsed star. Moreover, once this has been accomplished, we shall use the expressions so obtained for an evaluation of the corresponding moments \( A_{2m} \) of the light curves, in the form of associated series (terminating, or infinite) which converge to the desired results.

In doing so we shall find that while a mathematical description of the light changes previously discussed in the time-domain (cf. Kopal, 1959; Chapter IV) prominently featured Appell's generalization of the hypergeometric series in two variables of the form \( F_1 \), in the frequency domain followed at present a similar role will be played by Appell's functions of the type \( F_4 \), which will make their appearance on the frequency-domain stage almost from the beginning of our analysis, and will accompany us throughout the rest of the present paper.

2. Loss of Light during Eclipses

Specific cases of explicit forms of the associated alpha-functions \( \alpha_l^0(h, k) \) in terms of the respective parameters have been evaluated before: that corresponding to \( l = 0 \) (i.e., to uniformly bright discs) is trivial; and all those for which \( l \) is an even integer can likewise be evaluated in a closed form in terms of radicals and inverse trigonometric functions. However, for odd values of \( l \), closed-form expressions for \( \alpha_l^0 \) can be established only in terms of elliptic integrals of the first and second kind, with variable limits (of. Kopal, 1942). Moreover, different types of eclipses (partial, total, or annular; occultations or transits) called for the use of different limits of integration; so that different programs are required to compute the \( \alpha_l^0 \)'s in each case.

In the frequency-domain approach – where we define the loss of light during eclipse as a cross-correlation of two circular apertures in terms of Hankel transforms – most specious features of the problem inherent in the time-domain approach naturally disappear; and Equation (1.1) defines \( \alpha_l^0 \) as a function which is continuous and (at least piecewise) differentiable at any phase of the eclipse as well as outside of them; and the aim of the present section will be to demonstrate by use of convergent expansions which are easy to construct or program for automatic computation.

In order to do so, let us change over, on the right-hand side of Equation (1.4), to a new variable \( y \), defined by

\[ x = \frac{r_2 y}{r_1 + r_2}, \]  

(2.1)