CHANDRASEKHAR'S PERTURBATION METHOD-ORIENTED THEORIES UP TO THIRD ORDER FOR UNIFORMLY AND DIFFERENTIALLY-ROTATING POLYTROPIC STARS: ERROR-REMOVING TECHNIQUES

V. S. GERONYANNIS
Astronomy Laboratory, Department of Physics, University of Patras, Greece

and

F. N. VALVI
Department of Mathematics, University of Patras, Greece

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Abstract. In this paper we analyse error-removing techniques used without sufficient theoretical support in a previous paper, where Chandrasekhar's higher-order perturbation theories were developed for either uniformly or differentially-rotating polytropic stars.

1. Introduction

In two previous papers (Geroyannis and Antonakopoulos, 1981, hereafter referred to as P1; Geroyannis et al., 1979, hereafter referred to as P2) we extended the classical Chandrasekhar's first-order perturbation theory (Chandrasekhar, 1933a, b, c; see also Chandrasekhar and Lebovitz, 1962, hereafter referred to as CL), abbreviated FOT, up to terms of third order in the perturbation parameter \( v \) (P1, Equation (2.15)). In P1, in particular, we developed the third-order theory, abbreviated (TOT), on the basis of the second-order theory, abbreviated (SOT), which was developed in P2. Both (SOT) and (TOT) treat of the structure distortion of a polytropic star owing to either uniform or differential rotation. Moreover, in a recent paper (Geroyannis and Valvi, 1986, hereafter referred to as P3) we derived results from numerical implementation of (TOT), that we extensively compared with respective numerical results of previous work on rotating polytropes.

In the present paper we analyse some significant techniques used in P3 to remove, eliminate, or reduce the errors involved in the numerical determination of (i) the differential rotation functions, and (ii) the associated Emden functions of first, second, and third order.

A standard practice, applied to all the subsequent numerical integrations of differential equations, is the division of the integration interval

\[ I \equiv [0, \zeta_E], \quad \zeta_E = \xi_1 + E, \quad (1.1) \]

\( \xi_1 \) being the first root of the Lane-Emden function \( \theta(P1, \text{Equations (2.4) and (2.5)}) \) and

\[ \theta \]
E a small quantity, into \( N \) subintervals

\[
I_j = [\xi_j, \xi_{j+1}], \quad j = 0, 1, 2, \ldots, N - 1,
\]

with common length

\[
H = \frac{\xi_E}{N} = \xi_{j+1} - \xi_j.
\]

Hence, a point-mesh

\[
\Xi = \{\xi_j | \xi_j = jH, \quad j = 0, 1, 2, \ldots, N\},
\]

is set up, which is of particular significance for our method, since the solutions of all the functions and their first derivatives, corresponding to this mesh, are stored into individual arrays; and these functions can be approximated, whenever it is subsequently required, by any numerical method based upon equally spaced function tables.

In fact, this paper can be considered as parallel investigation to P3 in the sense that these two investigations are complement to each other. We notice, however, that some further problems, stated in P3, remain open for subsequent consideration and discussion.

Throughout this paper, parenthesized FOT, SOT, TOT, i.e., (FOT), (SOT), and (TOT) will denote our own work either theoretical or numerical.

2. Functions of Differential Rotation

To study systematically the effects of a gradually increasing differential rotation, we introduce the quantity \( F_r, 0 < F_r < 1 \), called 'reduction factor' or simply 'strength' of differential rotation. By this quantity, we assign new values

\[
b^* = F_r b_j
\]

(2.1)

to the nonuniformity parameters \( b_j \) (Clement, 1967, hereafter referred to as C1; Equation (10), Table I). The limiting differential rotation state represented by the value \( F_r = 0 \) is a uniform rotation. While the limiting state with \( F_r = 1 \) is that giving the sharpest pattern of differential rotation.

In terms of the new nonuniformity parameters \( b^*_j \), the differential equations for the auxiliary radial functions \( \psi_{jk} \) (C1, Equation (A2)) obtain the form

\[
\frac{d\psi_{jk}}{d\xi} = \frac{1}{\xi} [1 - (k + 1)\psi_{jk}] - 2b^*_j \xi \psi_{jk}, \quad j = 1, 2, 3, \quad k = 0, 2, 4, 6.
\]

(2.2)

By numerical integration of these differential equations under initial conditions \( \psi_{jk}(\xi = 0) = 1/(k + 1) \), we get the functions \( \psi_{jk} \); and in terms of the latter, the expansion radial functions \( \pi_i, \chi_i, \phi_i \) are given as follows (C1, Equations (A4)–(A6))

\[
\pi_i(\xi) = (2i + 1) \sum_j a_j \sum_k p_{jk} \psi_{jk}(\xi),
\]

(2.3)