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Abstract. The effect of Hall currents and collision with neutrals on the instability of a horizontal layer of a self-gravitating partially-ionized plasma of varying density have been studied. It is assumed that the plasma is permeated by a variable horizontal magnetic field stratified vertically. A variational principle is shown to characterize the problem. By making use of the existence of the variational principle, proper solutions have been obtained for a semi-infinite plasma in which density has a one-dimensional (exponential) vertical stratification. The dispersion relation has been derived and solved numerically. It is found that the collisions with neutrals have a stabilizing influence while Hall currents have a destabilizing influence.

1. Introduction

The problem of the instability of a magnetized plasma of variable density is of importance in several geophysical and astrophysical situations such as in the formation and mixing of clouds, heating of solar corona, stability of stellar atmospheres in magnetic fields and in theories of sunspot magnetic fields. This stability problem has attracted the attention of several researchers in the past and a comprehensive account of these investigations under varying physical assumptions has been given by Chandrasekhar (1961), in his monograph on hydrodynamic and hydromagnetic stability. The importance of such a stability in astronomical context has also been given by Ognesyan (1960a, b) and Shafranov (1960).

In the dynamics of interstellar matter and in several other astrophysical situations the Hall currents are of importance. Several researchers (e.g., Hosking, 1968; Singh and Tandon, 1969; Ariel, 1970; Bhowmick, 1972) have studied the effects of Hall currents on different problems of plasma instabilities. They have all pointed out that Hall effects are, in general, destabilizing. Bhatia (1974a, b) has investigated the effect of Hall currents on the stability of an incompressible self-gravitating stratified plasma of finite depth. There the plasma is assumed to be permeated by variable horizontal magnetic field and study has been carried out for the longitudinal mode of wave propagation. The Coriolis force are also of importance in geophysical and astrophysical situations. More recently, Alka Gupta and Bhatia (1991) have investigated the combined influence of Hall currents and Coriolis forces on the instability of a self-gravitating plasma of varying density for longitudinal mode of propagation when the plasma is permeated by variable horizontal magnetic field. In the absence of Hall currents the same problem has been investigated by Bhatia and Chonkar (1985) for the transverse mode of propagation wherein the effects of viscosity have also been considered. All these investigations have been carried out for a fully-ionized plasma.
In cosmic physics there are several situations such as in chromosphere, solar photospheres, and in cool interstellar clouds, the plasmas are frequently not fully-ionized but may instead be partially-ionized so that the interaction between the ionized fluid and the neutral gas becomes important. Alfvén (1954) has pointed out the importance of such collisions between ionized fluid and neutral gas on the ionization rate in these regions. Lehnert (1970) has considered the stability of a plasma interacting with neutral gas. Recently, Mehta and Bhatia (1988) have studied the instability of a horizontal layer of a self-gravitating partially-ionized plasma in the presence of effects of Hall currents, FLR, and ion viscosity when the plasma is permeated by vertical magnetic field.

It would, therefore, be of importance to examine the effects of Hall currents on the stability of a self-gravitating stratified partially-ionized plasma of finite depth in a variable horizontal magnetic field. This aspect forms the basis of this paper wherein both the magnetic field and density of plasma are exponentially stratified along the vertical direction.

2. Perturbation Equations

We consider the horizontal strata of a self-gravitating partially-ionized plasma of variable density moving in a horizontal variable magnetic field. We assume that the two components of the partially-ionized plasma (the ionized fluid and the neutral gas) behave as continuums and that their steady-state velocities are equal. Furthermore, we assume that the magnetic field interacts only with the ionized component of the plasma and that the frictional force of the neutral gas on the ionized fluid is of the same order as the pressure gradient of the ionized fluid.

Under the above assumption the relevant linearized perturbation equations governing the two components of the partially-ionized plasma are:

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\begin{align*}
\frac{\partial \mathbf{u}}{\partial t} & = - \nabla \delta \rho + (\nabla \times \mathbf{h}) \times \mathbf{H} + (\nabla \times \mathbf{H}) \times \mathbf{h} + \delta \rho \nabla \phi + \\
& \quad + \rho \nabla \delta \phi + \rho_d v_c (\mathbf{u}_d - \mathbf{u}) , \\
\frac{\partial \mathbf{u}_d}{\partial t} & = v_c (\mathbf{u}_d - \mathbf{u}) , \\
\frac{\partial \mathbf{h}}{\partial t} & = \nabla \times (\mathbf{u} \times \mathbf{H}) - \frac{1}{Ne} \nabla \times [(\nabla \times \mathbf{h}) \times \mathbf{H}] - \frac{1}{Ne} \nabla \times [(\nabla \times \mathbf{H}) \times \mathbf{h}] , \\
\frac{\partial}{\partial t} \delta \rho + (\mathbf{u} \cdot \nabla) \rho & = 0 , \\
\nabla^2 \delta \phi & = - G \delta \rho , \\
\nabla \cdot \mathbf{u} & = 0 \quad \text{and} \quad \nabla \cdot \mathbf{h} = 0 ;
\end{align*}
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