ISOTROPIZATION IN BRANS–DICKE–BIANCHI TYPE-V

(Letter to the Editor)

ENRIQUE GUZMÁN
Universidad Autónoma Metropolitana, México, D.F.

(Received 30 October, 1991)

Abstract. In this paper we find that from the exact solution for Bianchi type-V in the Brans–Dicke theory with $\gamma = 0$ the Hubble parameters are the same for $\phi \to \infty$, so that the Universe will be isotropized.

1. Introduction

Amongst the various modifications of the General Relativity (GR) theory the Brans–Dicke (BD) theory is treated most seriously (Brans and Dicke, 1961). The Bianchi type-V model is interpreted as the generalization of the isotropic ($k = -1$) Friedmann–Robertson–Walker (FRW) open universe (see, for example, Zel’dovich and Novikov, 1983). In this paper we have found the general vacuum solutions for Bianchi type-V in the BD-theory, in one of such solutions we have found the inflationary behaviour, it is shown that for the special $\gamma = 0$ case there is no anti-gravity ($\phi > 0$, $\phi \sim G^{-1}$, $G =$ gravitation 'constant') which disagrees with other physical solutions found in previous papers (Chauvet and Guzmán, 1986). General solutions for vacuum isotropic FRW ($k = +1, 0, -1$) case in the BD-theory have been found by O’Hanlon and Tupper (1972), Chauvet (1983), Cerveró and Estévez (1983), and Lorenz-Petzold (1984), the vacuum general solution for Bianchi type-I was given by Ruban and Finkelstein (1972) and Belinskii and Khalatnikov (1972). The Bianchi type-II general vacuum solution with total anisotropy ($R_1 \neq R_2 \neq R_3$) has been obtained by us recently (Guzmán, 1991). Finally, by use of our method we can find the corresponding general Bianchi type-V solution in the GE-theory first given by Joseph (1966).

2. Equations and their Solutions

The metric for Bianchi type-V is

$$\text{d}s^2 = -\text{d}t^2 + R_1^2\text{d}x^2 + \exp\left(-2bx\right)R_2^2\text{d}y^2 + \exp\left(-2bx\right)R_3^2\text{d}z^2. \tag{1}$$

The field equations for BD–Bianchi type-V vacuum are

$$\left(R_i/R_i\right)^- + \left(\dot{R}^3/R^3\right)\left(R_i/R_i\right) + \left(\dot{\phi}/\phi\right)\left(R_i/R_i\right) = 2b^2/R_i^2, \tag{2}$$

$$2\left(\dot{R}_1/R_1\right) = \left(\dot{R}_2/R_2\right) + \left(\dot{R}_3/R_3\right), \tag{3}$$
\[
\begin{align*}
(\hat{R}_1/R_1)(\hat{R}_2/R_2) + (\hat{R}_1/R_1)(\hat{R}_3/R_3) + (\hat{R}_2/R_2)(\hat{R}_3/R_3) - (w/2)(\ln \phi)^2 + \\
+ (\ln R^2)'(\ln \phi)' = 3b^2/R_1^2, \\
(R^3\phi)' = 0;
\end{align*}
\]
where \( R_i = R_i(t) \) \((i = 1, 2, 3)\) are the cosmic-scale factors \( R^3 = R_1R_2R_3 \). \( \phi = \phi(i) \) is the BD-scalar field and \( (\cdot)' = d/dt \), \( b \) is a constant. If we introduce the new variable \( d\phi = (1/R^3)\,dt \), Equations (2)--(5) are given by
\[
\begin{align*}
(\ln R_i)'' + (1/\phi)(\ln R_i)' &= 2b^2R^6/R_1^2, \\
2(\ln R_1)' &= (\ln R_2)' + (\ln R_3)', \\
(\ln R_1)'(\ln R_2)' + (\ln R_1)'(\ln R_3)' + (\ln R_2)'(\ln R_3)' - w/2\phi^2 + \\
+ (\ln R^3)'/\phi &= + 3b^2R^6/R_1^2,
\end{align*}
\]
where \( (\cdot)' = d/d\phi \). If we add (6) with \( i = 1, 2, 3 \) to eliminate the curvature term, we obtain by integration
\[
\begin{align*}
R_1 &= R_3\phi^k, \\
R_1 &= R_2\phi^z,
\end{align*}
\]
where \( k \) and \( z \) are constants. From Equation (7) we have
\[
k + z = 0.
\]
By substitution of (9) and (10) into (8) we obtain an equation only for \( R_1 \)
\[
2(\ln R_1)'^2 + 2(\ln R_1)'/\phi + \gamma/\phi^2 = 2b^2R^6/R_1^2,
\]
where \( \gamma \) is given by
\[
\gamma = (2\alpha k - w)/3.
\]
From (6) with \( i = 1 \) and (12) we obtain
\[
(\ln R_1)'' - 2(\ln R_1)'^2 - (\ln R_1)'/\phi - \gamma/\phi^2 = 0.
\]
By introducing the new variable \( d\eta = (1/\phi)\,d\phi \), where \( (\cdot)' = d/d\eta = \phi(\cdot)' \) into Equation (14) we obtain
\[
(\ln R_1)'' = 2(\ln R_1)'^2 + 2(\ln R_1)' + \gamma.
\]
Knowing that \( (\ln R_1)'' = (\ln R_1)' d(\ln R_1) \), by defining \( x = \ln R_1 \) we obtain
\[
dx = \dot{x} d\dot{x}/2\dot{x} + \gamma.
\]
The general solutions of Equation (16) are: for the \( 1 > 2\gamma \) case
\[
R^4_1 = R_{10}(\dot{x}^2 + \dot{x} + \gamma/2)(2\dot{x} + 1 - \sqrt{1 - 2\gamma})(2\dot{x} + 1 + \sqrt{1 - 2\gamma})^{-1/\sqrt{1 - 2\gamma}},
\]