HYDROMAGNETIC SURFACE WAVES IN A COMPRESSIBLE MEDIUM

(Letter to the Editor)

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Abstract. The dispersive characteristics of hydromagnetic surface waves along a plasma-plasma interface when the upper fluid moves with a uniform velocity is discussed. The region of propagation of these waves is shifted above or below depending on whether the basic velocity (uniform) $U \geq 0$.

The study of hydromagnetic waves in inhomogeneous magnetic fields is very important for delimiting various plasma effects in the solar atmosphere. A discontinuity in the Alfvén speed along an interface gives rise to hydromagnetic surface waves. There have been various studies on hydromagnetic surface waves both for the incompressible (Uberoi and Somasundaram, 1980; Roberts, 1981a, b) and compressible media (Wentzel, 1979; Roberts, 1981a, b; or Somasundaram and Uberoi, 1982).

Uberoi and Satya Narayanan (1986) have studied the dispersion equation for the hydromagnetic surface waves along the interface between two compressible plasma media when the magnetic field across the interface vary both in magnitude and direction.

In this paper we discuss how the propagation characteristics of hydromagnetic surface waves are affected when the fluid above the interface moves with a uniform velocity $U$.

We are interested in the behaviour of hydromagnetic surface waves in a medium in which the non-uniformity takes the form of a single magnetic interface, the field $B(x)$ being uniform but of different magnitude on either side of the interface $x = 0$ so that $B_0(x) = B_{01}$ for $x < 0$ in medium 1 with density $\rho_{01}$ and $B_{02}$ for $x > 0$ in medium 2 with density $\rho_{02}$. Medium 2 is assumed to move with a uniform velocity $U$.

For small perturbations of the form

$$f(x, y, z, t) \sim f(x) \exp i(ly + kz + \omega t),$$

the linearized MHD equations on solving and applying the boundary conditions at the interface yield the dispersion relations as

$$\varepsilon_1(k, \omega, U) (m_2^2 + l^2)^{1/2} + \varepsilon_2(k, \omega, U) (m_2^2 + l^2)^{1/2} = 0,$$

where $\varepsilon_{1,2}$ are the low-frequency dielectric constants of the magnetised plasma given as

$$\varepsilon_{1,2}(k, \omega, U) = k^2 B_{01,2}^2 \mu - \rho_{01,2} \omega^2 - \rho_{01,2} \omega U$$

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and \((m_{1,2}^2 + l^2)^{1/2}\) are the decaying constants with

\[
m_{1,2}^2 = \frac{(k^2c_{1,2}^2 - \omega^2 - Uk\omega)(k^2V_{1,2}^2 - \omega^2 - Uk\omega)}{[c_{1,2}^2k^2V_{1,2}^2 - (\omega^2 + uk\omega)(c_{1,2}^2 + V_{1,2}^2)]},
\]

where

\[
c_{1,2} = \left(\frac{p_{01,2}}{\rho_{01,2}}\right)^{1/2}
\]

and

\[
v_{A,1,2} = \left(\frac{B_{01,2}^2}{\mu\rho_{01,2}}\right)^{1/2}
\]

are the acoustic and Alfvén speeds in the two media with \(p_{01,2}\) as equilibrium hydrodynamic pressure.

Equation (1) reduces to the dispersion relation discussed by Somasundaram and Uberoi (1982) when \(U = 0\). For the incompressible case \(c_{1,2} \to \infty\) so that \(m_{1}^2 = m_{2}^2 = k^2\). The decaying constants are the same in both the cases with and without the upper fluid moving with a uniform velocity \(U\) for an incompressible plasma. However, they are different in the compressible case.

In finding the roots of Equation (1), which represents the possible modes of surface wave propagation, it should be noted that it will have real roots only when \(\varepsilon_1\) and \(\varepsilon_2\) are of opposite signs and for an exponentially decaying solution the terms \(M_{1,2}^2 + \tan^2\theta\) should both be positive, where \(M_{1,2} = m_{1,2}/k\) and \(\tan \theta = l/k\).

From Equation (2) it can be shown that \(\varepsilon_1\) and \(\varepsilon_2\) are of opposite signs only when

\[
\min \left(\frac{U}{2} + \sqrt{V_{A1,2}^2 + \frac{U^2}{4}}\right) < \frac{\omega}{k} < \max \left(\frac{U}{2} + \sqrt{V_{A1,2}^2 + \frac{U^2}{4}}\right);
\]

when \(U = 0\), Equation (3) reduces to

\[
\min(V_{A1,2}) < \frac{\omega}{k} < \max(V_{A1,2}).
\]

This relation was derived by Somasundaram and Uberoi (1982).

The condition that the expression \(M_{1,2}^2 + \tan^2\theta\) are to be positive again divides the \((\omega/kv_{A1}, c_{1}/v_{A1})\) plane into various regions. This region in which the wave propagation takes place is that in which relation (3) and \(M_{1,2}^2 > 0\) both hold good. Comparing the region of propagation obtained by Somasundaram and Uberoi (1982), with the present, we see that there is a shift above or below depending on whether \(U\) is \(\geq 0\). However, there is a significant change in the phase velocity of both the fast and slow magnetoacoustic modes when the dispersion relation is solved as the roots of a fifth-order polynomial in \((\omega/kv_{A1})^2\) for low velocities of \((c_{1}/v_{A1})\). We skip the details for the sake of brevity.