THE MOTION OF A SATELLITE IN A RING POTENTIAL

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Abstract. This investigation presents the orbital elements of a satellite moving in a circular ring potential. The ring is considered to be of infinitesimal thickness and of unit radius. The components of the perturbing accelerations due to the ring potential have been substituted into the Gauss form of Lagrange's planetary equations to yield the first-order approximations. The elements of the orbit have been expressed by means of Hansen coefficients. The results include the effects produced by the 2nd, 4th, 6th, and 8th spherical harmonics. Due to their importance we present separately the secular terms from the periodic ones. The general expressions for the orbital elements can be easily extended to include the effects produced by any other higher harmonic.

List of Symbols

\( x \)
- semi-major axis;

\( C_{jk}^{u} (u, \omega) \)
- cosine functions of \( u \) and \( \omega \);

\( e \)
- eccentricity of the orbit;

\( f \)
- \( \sin^2 \iota \);

\( \iota \)
- inclination of the orbit;

\( M \)
- mean anomaly;

\( n \)
- mean motion;

\( p \)
- semi-latus rectum of the orbit;

R, S, and W
- components of the perturbing acceleration;

\( r \)
- magnitude of position vector;

\( S_{jk}^{\nu} (u, \omega) \)
- sine functions of \( u \) and \( \omega \);

\( T \)
- time of periapse passage;

\( u \)
- argument of latitude;

\( U \)
- gravitational potential;

\( V \)
- perturbing potential;

\( \mu \)
- \( G(M_r + m) \) (gravitational constant times the sum of the masses of ring and satellite);

\( \nu_{n,k} \)
- coefficients of \( R \) component of disturbing acceleration (functions of \( f \));

\( \sigma_{n,k} \)
- coefficients of \( S \) and \( W \) components of disturbing acceleration (functions of \( f \));

\( \chi \)
- mean anomaly at time \( t = 0 \);

\( X_{0,m}^{n} \)
- zero-order Hansen coefficients;

\( \omega \)
- argument of periapse;

\( \Omega \)
- longitude of the ascending node.

1. Introduction

The solar system, in addition to the Sun, the major planets and their satellites consists of a wide variety of additional material such as planetary rings, minor planets, comets, and an abundance of assorted other debris.

Since the discovery of the rings of Uranus in 1977 (Elliott et al., 1977) and the confirmation of their existence (Millis et al., 1977; Bhattacharyya and Kuppuswamy,
1977; Bhattacharyya and Bappu, 1977; Bappu et al., 1979), no theory successfully explaining the peculiar features of these objects has been forthcoming. Additional measurements of the dimensions, orientation, and optical thickness of the individual rings are clearly needed.

In 1979 the Voyager 1 fly-bys of Jupiter and discovers its ring. Voyager missions reveal the detailed and surprising structure of Saturn's rings. This ring system is not only one of the most striking objects in the sky, but also one of the most scientifically interesting and currently one of the least well understood. The spokes of Saturn's rings are attributed to magnetohydrodynamical effect (Porco and Danielson, 1982).

Uranus has about 10 ringlets. Some of them are elliptical and some others are inclined to the Uranian equator. Saturn's rings at the extreme are composed of thousands of individual ringlets, extended well beyond the Roche limit. Every possible mechanism has been invoked in order to explain some part of the unusual, bizarre and fascinating characteristics of these objects.

A detailed study of the interaction between the satellites and the rings could lead to a better understanding of the planetary rings. In the case of the Saturnian ring near 1, 29R_s (R_s = Saturn radius) it appears to be well-understood, due to a recent analysis by Porco et al. (1984).

Secular perturbations for the orbital element $\bar{\omega} = \Omega + \omega$, in the case of an elliptical ring, have been presented by Greenberg (1981).

2. Equations of the Problem

In order to present the analytical expressions for the perturbations of a satellite moving in a circular ring potential, we employ six independent parameters or elements. These elements specify the orbit of the satellite in space and will be considered to be the semi-major axis $a$, the eccentricity $e$, the inclination $i$ to the ring, the longitude of the ascending node, the argument of periapse $\omega$ and $\chi$ the mean anomaly at time $t = 0$. The element $\chi$ is connected to the mean anomaly $M$ by the expression

$$\chi = M - nt = -nT,$$

where $n$ is the mean motion and $T$ is the time of periapse passage.

If $R, S$, and $W$ are the components of the disturbing acceleration, then the Gauss form of Lagrange's planetary equations, using $u$ as independent variable becomes (Zafiropoulos and Kopal, 1982a)

$$\frac{dz}{du} = \frac{2a^2}{\mu p} \{r^2e \sin(u - \omega) R + rp S\},$$

$$\frac{de}{du} = \frac{r^2}{\mu p} \{ p \sin(u - \omega) R + (r + p) \cos(u - \omega) S + er S\},$$

$$\frac{d\Omega}{du} = \frac{r^3 \sin u}{\mu p \sin i} W,$$