MAGNETO-GRAVITATIONAL INSTABILITY AND SUSPENDED PARTICLES

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Abstract. The gravitational instability of an infinite homogeneous and infinitely conducting self-gravitating gas-particle medium in the presence of a vertical magnetic field and suspended particles is considered. It is found that in the presence of suspended particles and magnetic field, Jeans' criterion determines the gravitational instability.

1. Introduction

A comprehensive account of the gravitational instability of an infinite homogeneous self-gravitating medium, as discovered by Jeans (1902), has been given by Chandrasekhar (1961). He has found that a uniform magnetic field do not alter Jeans' criterion for instability.

Sharma et al. (1976) have studied the effect of suspended particles on the onset of Bénard convection in hydromagnetics. They have found that the effect of suspended particles is to destabilize the layer and that the magnetic field has a stabilizing influence. In another study, Sharma (1975) studied the effect of suspended particles on the gravitational instability of an infinite homogeneous gas-particle medium.

The object of the present investigation is to study the gravitational instability of an infinite homogeneous and infinitely conducting self-gravitating gas-particle medium in the presence of magnetic field and suspended particles. This aspect forms the subject matter of the present paper. The presence of suspended (or dust) particles in the gas in astronomical contexts is more realistic and a reconsideration of the magneto-gravitational instability problem in the presence of suspended particles is certainly called for. This aspect forms the subject matter of the present paper.

2. Perturbation Equations

Let $u(u, v, w)$, $h(h_x, h_y, h_z)$, $\delta \varrho$, $\delta p$ and $\delta U$ denote respectively the perturbations in gas velocity, magnetic field $H(0, 0, H)$, density $\varrho$, pressure $p$ and gravitational potential $U$; $N(\bar{x}, t)$ and $v(\bar{x}, t)$ denote the number density and velocity field of the particles, $K = 6\pi \varrho \eta$, $\eta$ being the particle radius and $v$ the kinematic viscosity of the gas, is a constant and $\bar{x} = (x, y, z)$. Let $c$ and $G$ denote the velocity of sound in the medium and the constant of gravitation. Then the linearized perturbation equations governing the motion of the gas-particle medium are
\[
\frac{\partial \mathbf{u}}{\partial t} = -\frac{1}{\rho} \nabla \delta p + \nabla \delta U + \frac{KN}{\rho} (\mathbf{v} - \mathbf{u}) + \nu [\nabla^2 \mathbf{u} + \frac{1}{2} \nabla (\nabla \cdot \mathbf{u})] + \\
+ \frac{1}{4\pi \mu} \left( \nabla \times \mathbf{h} \right) \times \mathbf{H}, \tag{1}
\]

\[
\left( \tau \frac{\partial}{\partial t} + 1 \right) \mathbf{v} = \mathbf{u}, \tag{2}
\]

\[
\frac{\partial}{\partial t} \delta \rho = -\rho \nabla \cdot \mathbf{u}, \tag{3}
\]

\[
\delta P = c^2 \delta \rho, \tag{4}
\]

\[
\nabla \cdot \mathbf{h} = 0, \tag{5}
\]

\[
\frac{\partial \mathbf{h}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{H}), \tag{6}
\]

\[
\nabla^2 \delta U = -4\pi G \delta \rho, \tag{7}
\]

where \( mN \) is the mass of particles per unit volume and \( \tau = m/K \). In writing the linearized perturbation form (2) of the equation of motion for the particles, we have made use of the assumptions that the buoyancy force and the interparticle reactions are neglected (Scanlon and Segel, 1973). In writing Equation (1), use has been made of the Stokes' assumption that the bulk viscosity \( \lambda + \frac{2}{3} \rho v = 0 \).

### 3. Dispersion Relation

Assume that the perturbations of all the quantities are of the form

\[
\exp \left( ik_x x + k_z z + \sigma t \right), \tag{8}
\]

where \( k_x, k_z \) are the wave-numbers of the perturbation along the x- and z-axes and \( \sigma \) is the growth rate of the perturbation.

Using expression (8), Equations (1)–(7) give

\[
\begin{align*}
- \tau \sigma^3 - i \sigma^2 \left( 1 + \frac{3}{2} \nu k^2 \tau + \frac{KN\tau}{\rho} \right) + \frac{i}{3} \nu k^2 + \tau k^2 V^2 \sigma - \frac{ik^2 V^2}{2} \right) u = \\
- \sigma \left( \frac{ik_x}{k^2} \right) \Omega_j^2 (1 + i \sigma s), \tag{9}
\end{align*}
\]

\[
\begin{align*}
- \tau \sigma^3 - i \sigma^2 \left( 1 + \frac{3}{2} \nu k^2 \tau + \frac{KN\tau}{\rho} \right) + \sigma \left( \frac{3}{2} \nu k^2 + \tau k^2 V^2 \right) - \frac{ik^2 V^2}{2} \right) v = 0, \tag{10}
\end{align*}
\]

\[
\begin{align*}
- \tau \sigma^3 + i \sigma^2 \left( 1 + \frac{3}{2} \nu k^2 \tau + \frac{KN\tau}{\rho} \right) + \frac{i}{3} \nu k^2 + \tau k^2 V^2 \sigma \right) w = \\
= - \sigma \left( \frac{ik_z}{k^2} \right) \Omega_j^2 (1 + i \sigma s). \tag{11}
\end{align*}
\]