SUBSTRUCTURE IN CLUSTERS OF GALAXIES: A CLUE TO THE 'MISSING' MASS?

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Abstract. The influence of subclustering in rich clusters of galaxies is examined using results from numerical n-body experiments. It is found that, under some conditions, the standard virial theorem is satisfied. No physical missing mass is needed because its role is replaced by the gravitational energy of the subclustering. We find that, in the Coma cluster, this effect masquerades as a ‘missing’ mass about 7 times that of the physical mass, so that the apparent extant virial discrepancy \( M_{\text{vir}}/M \approx 8 \) in this cluster is explained.

1. Introduction

Substructure in rich clusters of galaxies is known to exist in the form of poor subclusters, groups, and multiple galaxy systems involving 2–7 components. Abell et al. (1969) measured parameters of subclustering in five of seven clusters they looked at, and De Vaucouleurs (1971) has given data on multiple galaxy systems. The missing mass problem in clusters of galaxies has become less acute over the years, with a tendency to concentrate attention on those rich clusters, like Coma \( (M_{\text{vir}}/M \approx 8; \) Tarter and Silk, 1974), where the ratio of virial mass to observed mass is not particularly excessive. Despite the existence of such cases, it is obvious that the missing mass problem still exists as an uncontroversial problem in the majority of clusters, since the mean value of \( M_{\text{vir}}/M \) for all well-observed clusters is about 30, with strongly documented cases up to \( M_{\text{vir}}/M \approx 100 \) (Karachentsev, 1966; De Vaucouleurs, 1968; Rood et al., 1970; Jackson, 1970). Discrepancies of order 100 are difficult to explain on the basis of any suggested forms of physical missing mass (Tarter and Silk, 1974). We wish to examine here the hypothesis that there is no ‘missing’ mass in rich clusters of galaxies, but that its effect is mimicked by the gravitational binding energy of galaxy configurations in subclusters. We hope to show that the standard virial theorem for a cluster is satisfied if the energy of a (reasonable) postulated substructure is taken into account. We find that the simple model we consider explains values of \( M_{\text{vir}}/M \approx 5–7 \) in rich clusters. It does not explain values \( M_{\text{vir}}/M > 10 \), but we consider that the effect we discuss nevertheless provides a valuable clue in the search for the ‘missing’
mass, and that a more extensive analysis of it could well provide the complete solution to the problem.

We make use of results from \textit{n}-body experiments in Section 2, arriving at an approximate expression for the gravitational binding energy of a cluster with substructure (Section 3). We tentatively accept the validity of the subclustering hypothesis, and in conclusion (Section 4) show that it removes the virial discrepancy for the Coma cluster. We consider extensions of the hypothesis and suggest an observational program to investigate the extent of subclustering in rich clusters of galaxies.

\textbf{2. \textit{n}-Body Experiments}

To calculate what part of the gravitational binding energy of a cluster is to be found in what levels of the substructure, it is necessary to know the distribution of galaxies as a function of their multiplicity $\mu$, the dominant binding energy $e_\mu$ and its spectrum $f_\mu(e_\mu)$ for subsystems formed of $\mu$ galaxies bound together. Information is available from relevant \textit{n}-body numerical experiments (Von Hoerner, 1960, 1963; Aarseth, 1963, 1966, 1974; Aarseth and Hills, 1972; Van Albada, 1968a, b; Heggie, 1975). An initially unclustered group of bodies evolves rapidly into a condensed system in which the mean density varies like $\rho \propto r^{-2}$ in the central regions with an extensive halo of particles having velocities far in excess of the roughly Maxwellian distribution that holds nearer the centre (Von Hoerner, 1960, 1963). Eventually, nearly all the gravitational energy of the system is found to be bound up in one close binary (60\% of cases) or triple system (30\% of cases) or $\mu=4$ system (10\% of cases) located at the centre of the configuration. The binding energy of the dominant 'hard' binary is $\approx 3/2$ the total (negative) energy of the entire group (Van Albada, 1968a) in cases where a $\mu=2$ system is left after the system attains dynamical equilibrium (in the other cases, one component of the triple is remote from a close pair; while for $\mu=4$, one of the component binaries contains nearly all the binding energy). About half of the initial $n$ bodies are lost before the equilibrium configuration is reached. It has been suggested (Ambartsumian, 1961; Wesson, 1974) that observed clusters of galaxies of some orders are being affected by the expansion of the Universe. Aarseth (1963, 1966) has performed \textit{n}-body experiments in this case, finding that a stable nucleus persists despite expansion at the Hubble rate, and that approximately the same phenomena occur as noted above for groups without systematic expansion, about half the galaxies being lost to the field before the attainment of a stable cluster configuration (see also Janin and Haggerty, 1974). As equilibrium is approached, hard binaries become harder (Aarseth, 1974). The binding energy of a binary is $e_2=m_1m_2/2a_2$ where $m_1$, $m_2$ are the component masses and $a_2$ is the semi-major axis (Plummer, 1915). In practical \textit{n}-body experiments, the number of binaries with energy in $(e_2, e_2 + de_2)$ is given by an empirical two-particle distribution function $f_2(e_2) \, de_2$, about the exact form of which there is controversy (Heggie, 1975). It may be easily verified that for the forms of $f_2(e_2)$ given by Heggie, the energy of $n_2$ binaries is always