Abstract. On the basis of numerical experiments on n-body binding energies we tentatively consider the following hypothesis: If the distance between two galaxies forming a binary system is $a_o$, and a cluster of galaxies that is substructured in a hierarchical fashion on all scales from $a_o$ upwards has a total mass $M$, then the total gravitational binding energy of the cluster is $\Omega_{TH} = -GM^2/2a_o$. As an explanation for 'missing' masses up to order 100 we test this hypothesis in three different ways, finding remarkable agreement with observation, with no need for physical missing mass.

1. Introduction

It has been demonstrated by comparison with n-body computer simulation that substructure can have an important effect on the binding energy of a cluster of galaxies that is subclustered on all scales (Wesson and Lermann, 1977; hereafter referred to as WLI). This phenomenon can explain values of $M_{VT}/M \lesssim 10$, but we have not been able to analytically investigate the idea in full because the substructure argument is basically a restatement of the general n-body problem. To proceed, we have considered relevant numerical data and extrapolated the argument of WLI to formulate the hypothesis stated in the next section. In Section 3 we test the hypothesis in three ways, finding extremely exact agreement with observation. Section 4 summarizes the conclusions.

2. A Hypothesis on the Binding Energy of a Substructured Cluster

On the basis of the four data to be taken as justification below, we tentatively consider the following hypothesis:

If the distance between two galaxies forming a binary system is $a_o$, and a cluster of galaxies that is substructured in a hierarchical fashion on all scales from $a_o$ upwards has a total mass $M$, then the total gravitational binding energy of the cluster is $\Omega_{TH} = -GM^2/2a_o$.

This hypothesis is based on the following pieces of evidence: (a) Hard binaries are unquestionably dominant as regards numerical n-body experiments. (b) Many
galaxies are in binaries (WLI). (c) A typical rich cluster has a density profile of the type \( q = q_0 r^{-\theta} \) with \( \theta \approx 2 \), and a power-law of this type has no scale-length in it. Over the years, the ‘sizes’ of rich clusters have grown as it has been realized that rich clusters do not, in fact, have ‘edges’ that can be defined by observation. The ‘size’ of the Coma cluster for example, has grown by about an order of magnitude since it was first seriously observed, until it is now believed to be of supercluster dimension (Chicarini and Rood, 1975). Therefore, by elementary dimensional analysis, if the gravitational binding energy is \( \Omega = -GM^2/2l \), where \( l \) is a characteristic length, it is difficult to justify putting \( l = R \) where \( R \) is the ‘radius’ of the cluster, as is usually done. On the contrary, if the cluster is substructured on all scales from a smallest scale \( a_g \) (identified as the distance between the components of binary galaxies) upwards, then one expects \( l = a_g (= 2a_2, \) where \( a_2 \) is the binary semi-major axis). The \( M \) to be used is the total mass out to some distance (which is in practice \( R \), the limiting radius as fixed by galaxies whose membership of the cluster can be established at that distance). \( M(R) \) is calculable as the mass out to a distance \( R \) defined via \( a_g \) by a certain number of levels of the hierarchy, and is the observed, as opposed to virial, mass. (d) In principle, there is a dimensionless number that can also appear in \( \Omega_{TH} = -GM^2/2a_g \), but a series of numerical experiments which we are in process of performing on calculations of \( \Omega = -G \sum_m m_i m_j r_{ij} \) suggest to us that \( \Omega_{TH} \) is correctly given as we have stated it. This is supported by other calculations (Henriksen and Wesson, 1976) dealing with a geometrical approach to hierarchical binding energy.

The hypothesis, as stated, cannot at this stage be proven, since no adequate treatment of the n-body problem is available, but we can proceed to test its reasonableness by comparison with astrophysical data. If the static virial theorem, \( 2T + \Omega_s = 0 \), as used with a smooth, unstructured background, is employed, it gives a mass \( M_{VT} = 2R\bar{v}^2/G \), where \( \bar{v}^2 \) is the mean square dispersion velocity and \( \Omega_s = -GM^2/2R \). We can thus predict, on the basis of the hypothesis given above, that

\[
\frac{2T}{\Omega_s} = \frac{M_{VT}}{M} = \frac{\Omega_{TH}}{\Omega_s} = \frac{R}{a_g}.
\]

We can compare the ratio of ‘missing’ mass to observed mass \( (\sim M_{VT}/M) \) as a function of the mean sizes \( (\bar{R}) \) of clusters with the relation (1), assuming that the mean distance \( (a_g) \) between the members of the component binary galaxies is fixed.

### 3. Three Tests of the Hypothesis

The relation (1) enables us to explain the following expected correlations: (i) A plot of \( \log_{10}(M_{VT}/M) \) versus \( \log_a \bar{R} \) for rich clusters should be a straight line and should have a slope unity (a logarithmic scale is used to accommodate all the data conveniently; the continued use of the static virial theorem with the length \( l = \bar{R} \) instead of \( a_g \) has provided many data on \( M_{VT}/M \)). An extensive set of values of \( M_{VT}/M \) has been given by De Vaucouleurs (1968), and these were employed by Rood et al. (1970)