MAGNETOGASDYNAMIC CYLINDRICAL SHOCK WAVE MODEL IN AN OPTICALLY THIN ATMOSPHERE

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(Received 11 February, 1987)

Abstract. Similarity solutions for one-dimensional unsteady isothermal flow of a perfect gas behind a magnetogasdynaminc shock wave including the effects of thermal radiation has been investigated in a uniform thin atmosphere. The flow is caused by an expanding piston and the total energy of the flow is assumed to be constant. Radiation pressure and energy have been neglected in comparison to radiation heat flux and the gas is assumed to be grey and opaque.

1. Introduction

Kopal and Lin (1951) have studied the propagation of shock waves in stellar interiors. Elliot (1960), Wang (1964), Helliwell (1969) have considered the problem of shock wave propagation with thermal radiation using the similarity method of Sedov (1959) in ordinary gas dynamics. A self-similar adiabatic flow behind a radiation-driven strong shock wave in uniform atmosphere has been studied by Wilson and Turcotte (1970). Rao and Ramana (1973) have also obtained the solutions of the self-similar adiabatic and isothermal flows in uniform atmosphere using the method of Laumbach and Probstein (1969). Sakashita and Morita (1977) have discussed the axially-symmetric explosion with thermal radiation in an exponential medium using the method of Lambach and Probstewn (1969). Elliot (1960) has studied self-similar solutions for spherical blast waves in air using Rosseland's diffusion approximations under the assumption of non-existence of heat flux at the centre of symmetry. Ojha (1972) studied the same problem including the effects of azimuthal magnetic field. Verma and Singh (1979), Ojha (1987) have discussed a similar piston problem with thermal radiation in magnetogasdynamics by using Helliwell's differential approximation for the equations of radiative transfer. By using Planck's diffusion approximation, Ray and Banerjee (1980) have obtained similarity solutions for a strong plane shock wave in a transparent grey gas of uniform density at very low pressure.

In the present case the flow behind a cylindrical shock wave in a uniform optically thin and grey medium including the effect of axial-magnetic field has been investigated by using the method of similarity solutions. It is assumed that the medium is non-viscous ideal gas of infinite electrical conductivity. The radiation pressure and energy have been neglected. The gas is assumed to be grey opaque and shock to be isothermal.

2. Fundamental Equations and Shock Conditions

The fundamental equations describing one-dimensional axially-symmetric flow of a perfect gas including the effects of radiation heat flux and axial-magnetic field are given.
by (cf. Greifinger and Cole, 1961)

\[
\frac{DP}{Dt} + \rho \left( \frac{\partial u}{\partial r} + \frac{u}{r} \right) = 0, \tag{2.1}
\]

\[
\frac{Du}{Dt} + \frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{1}{\rho} \frac{\partial}{\partial r} \left( \frac{B^2}{2\mu} \right) = 0, \tag{2.2}
\]

\[
\frac{DB}{Dt} + B \left( \frac{\partial u}{\partial r} + \frac{u}{r} \right) = 0, \tag{2.3}
\]

\[
\frac{D}{Dt} \left( \frac{p}{\gamma - 1} \right) + p \frac{D}{Dt} \left( \frac{1}{\rho} \right) + \frac{1}{\rho r} \frac{\partial}{\partial r} \left( Fr \right) = 0. \tag{2.4}
\]

In these equations, \( p, \rho, u, \) and \( B \) are the pressure, density, radial velocity of the gas, and axial-magnetic field. All the variables are functions of radial distance \( r \) and the time \( t \). \( \gamma \) and \( \mu \) are the ratio of specific heats and magnetic permeability.

Assuming local thermodynamic equilibrium and using Planck's diffusion approximation, we have

\[
\frac{\partial F}{\partial r} = 4\mu^*aT^4, \tag{2.5}
\]

where \( \mu^* \) is Planck's mean absorption coefficient; \( a \), the Stefan–Boltzmann constant; and \( T \), the absolute temperature. The equation of state for ideal gas is given by

\[
p = \Gamma \rho T. \tag{2.6}
\]

We take \( \mu^* \) as a power law function of the density and temperature (Wang, 1966) as

\[
\mu^* = \mu_0 \rho^{\alpha'} T^{\beta'}, \tag{2.7}
\]

where \( \mu_0, \alpha', \), and \( \beta' \) are constants.

The shock conditions are:

\[
\frac{\rho_0}{\rho_s} = 1 - \frac{u_s}{V}, \tag{2.8}
\]

\[
p_s - p_0 + \frac{1}{2\mu} \left( B_s^2 - B_0^2 \right) = \rho_0 u_s V, \tag{2.9}
\]

\[
\frac{B_s}{B_0} = 1 - \frac{u_s}{V}, \tag{2.10}
\]