THE RELATIVISTIC DYNAMICS OF A THIN SPHERICALLY SYMMETRIC RADIATING SHELL IN THE PRESENCE OF A CENTRAL BODY

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Abstract. We construct an idealized spherically symmetric relativistic model of an exploding object within the framework of the theory of surface layers in GR. A Vaidya solution for a radially radiating star is matched through a spherical shell of dust to a Schwarzschild solution. The (incomplete) equations for the motion of the spherical shell of dust and the radiation density of the Vaidya solution, as given by the matching conditions, are reduced to a first-order system and a general analysis of the characteristics of the motion is given. This system of differential equations is completed, adding a relation between the unknowns which represents the simplest way to avoid an unphysical singularity in the motion. The results of a numerical integration of the equations are presented in two cases which we think may have some relationship to stellar explosions. A comparative set of results for other solutions is also given, and some possible generalizations of the model are pointed out.

1. Introduction

It is always of great interest to construct solutions of the field equations of general relativity (GR) with direct relevance to particular problems in astrophysics. The study of the external gravitational field of a central body combined with a radiating spherical shell of dust is clearly of physical significance since it may correspond to an idealized model of an explosive event in the stage in which a shell has been formed and ejected from a collapsing core. This may be in consonance with a star model in which a highly evolved core implodes to nuclear densities while at the same time the outermost layers of matter are blown off at high speed. This explosive event may be associated with the birth of a neutron star or black hole (Colgate, 1975).

In this paper we develop a very simple model of an exploding object within the framework of the surface layer formalism in GR (see, e.g., Israel, 1966; Papapetrou and Hamoui, 1968). We construct a solution of the Einstein field equations by matching a Schwarzschild solution through a thin (but optically thick) spherical shell of dust to a Vaidya solution for a radially radiating star (Vaidya, 1951). The matching
conditions represent an incomplete set of coupled differential equations for the motion of the spherical dust shell and the radiation density of the Vaidya solution.

In Section 2 we present a brief summary of useful results of the theory of surface layers in GR and discuss the space–time metric and the equations of motion of our model. Section 3 is devoted to a reduction of the equations of motion to a first-order system and to the analysis of the general characteristics of the motion. We end the section with a derivation of the frequency shift of spectral lines and the luminosity of the exploding object as measured by a distant observer at rest. In Section 4 we complete the system of differential equations adding a relation between the unknowns which represents the simplest way to avoid an unphysical singularity in the motion as a function of the shell proper time. We choose in this way a particular family of solutions. The results of a numerical integration of the equations are presented in two cases which may correspond to idealized stellar explosion events. A comparative set of results for other solutions is also given. Finally, in Section 5 we point out some similarities of the solutions both from the theoretical and observational point of view and comment on some improvements and additions which could be made to future models.

2. The Space–Time Model

We give first a brief summary of useful results of the theory of surface layers in GR.* Let $M^+$ and $M^-$ be two manifolds having a common timelike boundary $\Sigma$, and let $K^+$ and $K^-$ be the second fundamental tensor of $\Sigma$ in $M^+$, $M^-$ respectively. The full space–time, $M^+ \cup \Sigma \cup M^-$ is denoted by $M$. The metric $g$ (signature $+2$) of $M$ is assumed to be of class $C^3$ in $M^+$ and $M^-$. The metric $g$ induces a unique Lorentzian metric $\tilde{g}$ of class $C^3$ in $\Sigma$. Explicit expressions for the components of $K^+$, $K^-$ are

$$K^{ab+} = -h^A_h^B N^c \delta^d_{;d} \quad \text{at } \Sigma; \quad h^A = \delta^A_a - N_a N^a,$$

(2.1)

where $\delta^d_{;d}$ indicates the covariant derivative with respect to the Riemannian connection of $M^+$ and $M^-$ respectively, and $N$ is the unit normal vector to $\Sigma$ (directed from $M^-$ to $M^+$). If the 3-tensor (defined on $\Sigma$) $\gamma = K^+ - K^-$ is nonzero, the hypersurface $\Sigma$ is called (the history of) a surface layer. A surface stress–energy tensor $S$ of the layer is related to $\gamma$ by

$$\delta \pi S = \gamma - \tilde{g} \tr \gamma,$$

(2.2)

expressing the junction conditions at $\Sigma$.

Let now the hypersurface $\Sigma$ be the history of a spherical shell of dust; $S = \eta u \otimes u$, where $\eta$ is the surface energy density and $u = \partial / \partial \tau$, with $\tau$ the proper time along the world-line of a dust particle. The spherical symmetry of $\Sigma$ means that its intrinsic

* We adopt the sign conventions of Misner, et al. (1973). Relativistic units in which $c = G = 1$ are used.