GLOBAL MOMENTUM LOSS IN A NON-EXPANDING UNIVERSE

(Letter to the Editor)

ERNST FISCHER

Philips GmbH, Forschungslaboratorium Aachen, Germany

(Received 26 September, 1991)

Abstract. Applying the basic concepts of general relativity to the global motion of a particle in a mass-filled universe leads to a loss of momentum relative to the rest frame of the Universe. This loss is caused by the different running times of the gravitational interaction quanta exchanged with masses in front and behind the moving particle, if the signal velocity is limited to the speed of light. Due to this 'gravitational viscosity' of space, the energy of photons will be reduced with the time, and thus with the distance of the emitting source. This red shift is superimposed on the Doppler shift in an expanding universe. A discussion of the limiting case of vanishing expansion leads to predictions about mass and radius of the Universe. The value of the mass density in such a steady-state universe must be about three times the closing density discussed in Big-Bang theories. The existence of the 'gravitational viscosity' casts severe doubts on all estimations of the age of the Universe derived from the red-shift data.

It is generally accepted that conservation of momentum and energy in closed systems is one of the corner stones of mechanics. Newton's first law states that any mass moves with constant velocity along a straight line in the absence of outer forces. Thus it appears as a natural consequence to assume that also in general relativity momentum is conserved for 'geodesic motion', that means for force-free motion along the 'straightest line' with constant gravitational potential. The existence of such a form of motion is not proposed by the theory itself, but has to be introduced as an additional assumption. As has been noted already by Einstein, conservation laws for energy and momentum alone do not exist, as these quantities can be transferred to the tensor potential of gravity. In this paper we will show that 'geodesic motion' in a mass-filled universe is not compatible with the assumption that the velocity of gravitational interaction is limited to the speed of light.

For this purpose we will study the behaviour of a test particle, moving with respect to a reference frame, given by a homogeneous mass-filled universe. We will study the effects in a quasi-Newtonian approximation, but with the constraints imposed by the general relativity:

(1) Gravitational interaction is limited to the speed of light.
(2) Mass or energy cause an intrinsic curvature of space. A spatially-homogeneous universe can thus be regarded as a three-dimensional surface of constant curvature.
(3) Lines of force between masses follow the geodesic lines connecting them.

To avoid confusion with effects stemming from time-dependence of the metric, we confine the discussion to the time-independent metric of a static Einstein universe. To
describe the motion on a surface of constant curvature it is convenient to use 4-dimensional spherical coordinates defined by

\[ x = r \cos \gamma \cos \theta \cos \phi, \quad y = r \cos \gamma \cos \theta \sin \phi, \]
\[ z = r \cos \gamma \sin \theta, \quad w = r \sin \gamma, \]

where \( x, y, z, \) and \( w \) is a set of four Cartesian coordinates. In this system we determine the force exerted on a test mass at \( P = (R, 0, 0, 0) \) by the mass contained in a volume element \( dV \) of the surface \( r = R \), the volume element being given by

\[ dV = R^3 \cos^2 \gamma \cos \theta \, d\gamma \, d\theta \, d\phi. \]  \hspace{1cm} (2)

The distance between the element and \( P \) as measured on a straight line through 4-dimensional space is given by

\[ r_p = \sqrt{(x - R)^2 + y^2 + z^2 + w^2} = \sqrt{2 - 2 \cos \gamma \cos \theta \cos \phi}. \]  \hspace{1cm} (3)

Measuring along the geodesic the distance is

\[ s = 2R \arcsin \frac{r_p}{2R} = 2R \arcsin \sqrt{\frac{1 - \cos \gamma \cos \theta \cos \phi}{2}}. \]  \hspace{1cm} (4)

The component of gravitational force at point \( P \) in some direction, defined by the unit vector \( \mathbf{e} \), is given by

\[ dF = G \rho m \frac{dV}{s^2} (e \cdot e), \]  \hspace{1cm} (5)

where \( \rho \) is the mass density, \( G \) the gravitational constant, and \( e \), the unit vector in direction of the geodesic at point \( P \). As all directions are equivalent, we can choose \( e \) in the direction of the \( y \)-coordinate. In this case the projection onto the direction of the force component is

\[ (e \cdot e) = \frac{\cos \gamma \cos \theta \sin \phi}{\sqrt{1 - \cos^2 \gamma \cos^2 \theta \cos^2 \phi}}. \]  \hspace{1cm} (6)

For a Universe with constant mass density we obtain the total force in the \( y \)-direction by integrating Equation (5) over all distances and directions to be

\[ F_y = G \rho m R^3 \int_{-\infty}^{\infty} \int_{-\pi/2}^{\pi/2} \int_{-\pi/2}^{\pi/2} \frac{\cos^3 \gamma \cos^2 \theta \sin \phi}{s^2 \sqrt{1 - \cos^2 \gamma \cos^2 \theta \cos^2 \phi}} \, d\gamma \, d\theta \, d\phi. \]  \hspace{1cm} (7)

The limits of integration over the angle \( \phi \) are set to \( \pm \infty \), as the lines of force may extend also beyond the reciprocal pole \((-R, 0, 0, 0)\).

For any mass at rest with respect to the frame of reference it is obvious that this integral is zero due to symmetry. In the Newtonian limit this holds also for a moving