POPULATION I PULSATING STARS

III. Period-Evolutionary Mass(-Colour) Relations

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Abstract. Evolutionary masses corresponding to various evolutionary phases of Population I pulsating stars (89 Delta Scuti variables and 155 classical cepheids) are interpolated in the systems of tracks of Iben (1967) and Paczyński (1970). The evolutionary masses are larger in the latter system than in the former one. The uncertainty of the evolutionary mass of a star is estimated, when various evolutionary phases are possible for this star (a smaller evolutionary mass corresponds to a later phase). Semi-empirical period–evolutionary mass–colour ($P$–$M_e$–$C$) and period–evolutionary mass ($P$–$M_e$) relations are derived for various modes, groups of stars, colour indices (and effective temperature), and evolutionary phases. For Delta Scuti stars, the uncertainty of evolutionary masses calculated from the $P$–$M_e$ relations for different modes, is estimated. The improvement of the evolutionary mass accuracy is estimated, when a $P$–$M_e$–$C$ relation is used instead of the corresponding $P$–$M_e$ relation. The theoretical and semi-empirical period ratios of radial pulsations derived from the $P$–$M_e$ relations for Delta Scuti stars, are compared. There is a relatively good agreement between the $P$–$M_e$ relations for the two types of Population I pulsating stars, but a 'gap' exists between them. The evolutionary masses of these stars are closer in the two systems of tracks and are derived with a relatively higher accuracy in comparison with their ages.

1. Introduction

Direct determinations of stellar masses are only possible in special cases (mainly in binary systems). In most cases, other methods for mass estimations have to be applied. For example, if the position of a star on the H–R diagram is known, then by interpolating the evolutionary tracks in a given system one may estimate both the age and the mass (so-called 'evolutionary mass') of this star (Tsvetkov, 1986a; hereafter referred to as Paper III). On the other hand, one may evaluate, for pulsating stars, the so-called 'pulsation mass' based mainly on the period–mean density relation $Q = P \sqrt{\rho/\rho_\odot} = P \sqrt{(M/M_\odot)(R_\odot/R)^3}$; the period $P$, radius $R$, and pulsation 'constant' $Q$ for a given mode of pulsations have to be known in order to determine the pulsation mass $M$. For the classical cepheids, the well-known problem for discrepancy between the evolutionary mass and the pulsation mass exists and many papers which have tried to solve the problem have been published in the last fifteen years. For the Delta Scuti stars, such a problem does not exist (e.g., Tsvetkov, 1977, 1982b), with the exception of several variables (Tsvetkov, 1984).

It is interesting to relate the two types of Population I pulsating stars – Delta Scuti variables and classical cepheids – in terms of their masses. In the present paper we investigate the evolutionary masses of these objects and derive semi-empirical period–evolutionary mass-colour and period–evolutionary mass relations. We considered the same stars as in Paper III, in which results for the stellar ages were represented. In the
present paper, as in Paper III, two systems of evolutionary tracks — of Iben (1967) and of Paczyński (1970) — are used.

Delta Scuti variables are investigated in Section 2. In Section 3, classical cepheids are studied and a comparison with Delta Scuti stars is carried out. The main results of this paper are summarized in Section 4.

2. Delta Scuti Variables

2.1. Evolutionary Masses

A way for interpolation of evolutionary tracks, in order to estimate ages and evolutionary masses of stars, is described in Paper III. For Delta Scuti stars, three successive evolutionary phases after the time of ZAMS are taken into account: the major core hydrogen burning (CHB) phase (phase 1); the overall contraction of the stars (phase 2); and the shell hydrogen-burning (SHB) phase (phase 3). On the other hand, the mode of radial pulsations (F, 1H, 2H, 3H) is allowed for when we obtain various relationships for these variables.

Evolutionary masses (in units of the solar mass) of the considered Delta Scuti stars are listed in Table I. As was noted in Paper III, the evolutionary tracks of Iben lie to the upper left of the H–R diagram in comparison with the corresponding tracks of Paczyński and, hence, the evolutionary masses of stars are smaller in the system of Iben. It follows from Table I that for a star in a given evolutionary phase, the ratio of the evolutionary masses \( \frac{M_e(\text{Iben})}{M_e(\text{Paczyński})} \) is, typically, 90–94%. Consequently, the uncertainty of the evolutionary mass estimation of a Delta Scuti star is small — not larger than 10% — when different systems of Population I tracks are used. In Paper III it was found that such an uncertainty of the ages is considerably larger and may achieve up to one order in some cases.

2.2. Period–Evolutionary Mass–Colour and Period–Evolutionary Mass Relations

In Paper III we found semi-empirical period–age–colour and period–age relations for Delta Scuti stars in the SHB phase, in both systems of tracks and various modes of radial pulsations. In the present paper we derive period–evolutionary mass–colour \((P-M_e-C)\) and period–evolutionary mass \((P-M_e)\) relations for the same variables. These relations connect a pulsation characteristic (period) with an evolutionary one (evolutionary mass).

We obtained by the least-squares method three kinds of semi-empirical \(P-M_e-C\) relations for the four lowest modes of radial pulsations:

\[
\log(M_e)_3 = a_1 \log P + b_1 \log T_e + c_1 ,
\]
\[
\log(M_e)_3 = a_2 \log P + b_2 (b - \gamma)_0 + c_2 ,
\]
\[
\log(M_e)_3 = a_3 \log P + b_3 \beta + c_3 .
\]