EXTERNAL RADIATION FIELD IN RAYLEIGH–CABANNES ATMOSPHERES WITH CONSTANT AND LINEAR SOURCES

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Abstract. The external field of radiation in Rayleigh–Cabannes atmospheres with constant and linear sources is found using the resolvent matrix approach. If the internal sources are constant the external field may be described by the X-, Y-, and H-matrices. For the case with linear sources we need the derivatives of these matrices with respect to angular variable. The respective scheme for their determination is given.

A set of integro-differential equations for the X- and Y-matrices is derived and solved numerically. Some relations between the moments of the H-matrix are given and a sample of results for external fields are provided.

1. Introduction

In this paper we find the external fields of radiation for homogeneous plane-parallel Rayleigh–Cabannes atmospheres with constant or linear axially-symmetric internal sources using the resolvent matrix approach. The respective solution for the scalar case may be expressed in the X-, Y-, and H-functions and their derivatives with respect to angular variable depending on whether the atmosphere is optically finite or semi-infinite (Viik et al., 1985). For the vector case we encounter a remarkable similarity, only in this time we have to do with respective matrices. As a result we have to find a way to obtain the numerical values of these matrices. For finite atmospheres we may use a procedure of determining the X- and Y-matrices based on the method of discrete ordinates (Viik, 1991). In the present paper we propose another way, namely closely following the respective derivation in scalar case we derive a set of matrix integro-differential equations which may be solved by discretizing the integral terms. For a semi-infinite atmosphere the spectrum of methods to find the H-matrix is much broader. The nonlinear integral equation for the H-matrix for pure Rayleigh scattering was solved iteratively first by Lenoble (1970) and Abhyankar and Fymat (1970, 1971). For the case of almost conservative scattering, which is most interesting, this method does not converge. Tables of the H-matrix for molecular (or Rayleigh–Cabannes) scattering have been given by Bond and Siewert (1971). They have used the Gauss–Seidel iteration which converges even for the most difficult case – conservative Rayleigh scattering but is rather time-consuming. Kriese and Siewert (1971) have proposed a rapidly converging scheme which incorporates the linear constraint thus uniquely specifying the solution (Pahor, 1968).

Lately, de Rooij et al. (1989) have elaborated a general iteration method to find the H-matrix which compares favourable with that of Kriese and Siewert (1971) since in the framework of this approach there is no need to solve the characteristic equation and
to find the eigenvectors. Quite independently this same scheme has been rediscovered by Ivanov (1991).

Last but not least we may find the $H$-matrix by using the method of discrete ordinates (Viik, 1991).

2. The Equation of Transfer

The equation of transfer for the axially-symmetric part of the radiation field in a homogeneous plane-parallel atmosphere with internal sources may be written (Chandrasekhar, 1960; Domke, 1972) as

$$
\mu \frac{\partial I(\tau, \mu, \tau_0)}{\partial \tau} = I(\tau, \mu, \tau_0) - A_1(\mu) s(\tau, \tau_0),
$$

(1)

where

$$
s(\tau, \tau_0) = \frac{1}{2} \lambda \int_{-1}^{+1} A_2(\mu') I(\tau, \mu', \tau_0) \, d\mu' + s_0(\tau).$$

(2)

In Equations (1) and (2) $I$ is the intensity vector in the $(l, r)$ representation; $\tau$, the optical depth measured from the upper boundary of the atmosphere; $\mu$, the cosine of the angle between the direction of travel of a photon and the positive $\tau$-axis, $\tau_0$ is the optical thickness of the atmosphere ($0 < \tau_0 \leq \infty$), $\lambda$ is the albedo of the single scattering ($0 < \lambda \leq 1$) and $s_0(\tau)$ is the Stokes's vector of the internal sources of radiative energy.

In the case of molecular scattering in the atmosphere the matrices $A_1$ and $A_2$ are defined in the way

$$
A_1(\mu) = \begin{pmatrix}
1 - \mu^2 & \mu^2 \\
0 & 1
\end{pmatrix},
$$

(3)

$$
A_2(\mu) = B A_1^T(\mu),
$$

(4)

$$
B = \frac{3}{4} c \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2} (1 - c) \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix},
$$

(5)

and

$$
c = \frac{2(1 - \rho_n)}{2 + \rho_n},
$$

where $\rho_n$ is the depolarization factor ($0 \leq c \leq 1$). The boundary conditions for Equation (1) are

$$
I(0, -\mu, \tau_0) = I(\tau_0, \mu, \tau_0) = 0.
$$

(6)