NONLINEAR PERIODIC AND ALGEBRAIC ION-ACOUSTIC 
SOLITARY WAVES IN A RELATIVISTIC PLASMA 

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Abstract. We derive a mixed modified Korteweg–de Vries (MK–dV) equation from a semi-relativistic ion acoustic wave with hot ions by the fluid approximation. The positive cubic nonlinearity of the mixed MK–dV equation give rise to the periodic progressive waves and the algebraic solitary waves. The periodic wave bears a series of solitary pulses, and the algebraic solitary wave reduces the rarefactive solitary wave in the limit of the particular boundary condition. These nonlinear wave modes explain, respectively, the periodic pulse of the potential and the rarefactive solitary wave of the fine structure observed in space.

1. Introduction

In a historical current of nonlinear wave studies on plasmas, solitary waves have been drawing attention by a lot of investigators. The propagation of solitary waves is important as it describes characteristic nature of the interaction of the waves and the plasmas. In the case where the velocity of particles is much smaller than that of light, ion-acoustic waves present the non-relativistic behaviours, but in the case where the velocity of particles approaches that of light, the relativistic effect becomes dominant.

The interplanetary space and the Earth's magnetosphere encompass a rich variety of plasma physical processes and nonlinear wave phenomena. Various kinds of nonlinear waves occur in relativistic plasmas and thus relativistic Langmuir and electromagnetic waves have been studied as subjects of laser-plasma interaction and space plasma phenomena. Actually high speed and energetic streaming ions with the energy from 0.1 to 100 MeV are frequently observed in solar atmosphere and interplanetary space. Nevertheless, relativistic ion-acoustic waves have not been so well investigated. When we assume that the ion energy depends only on the kinetic energy, such plasma ions have to attain very high velocity of relativistic order. Thus, by considering the weakly relativistic effect where the ion velocity is about $\frac{1}{10}$ of the velocity of light, we can describe the relativistic motion of such ions in the study of nonlinear interaction of the waves and the plasmas. Since the ion temperature is very high in solar flare (Smith and Brecht, 1985), solar wind (Lin et al., 1986) and interplanetary space (Lyons and Williams, 1984), the ratio of the ion temperature and the electron temperature is sometimes more than 1. In this situation, it appears that the relativistic and ion temperature effects play an important role to energetic ion-acoustic waves propagating in interplanetary space. Relativistic ion-acoustic solitary waves have been found by the Korteweg–de Vries (K–dV) equation (Nejoh, 1987). Interesting features such as the formation of the precursor of the ion-acoustic solitary waves, the long wavelength ion-oscillation modes

(Nejoh, 1987), and the association between relativistic ion modulation modes (Nejoh, 1988) are discussed as astrophysical phenomena. The author shows a new spiky solitary wave and an explosive mode with negative potential of ion-acoustic waves obliquely propagating in a magnetized plasma (Nejoh, 1990) and also shows a new type of modulationally unstable relativistic ion-acoustic solitary wave (Nejoh, 1992). Although the relativistic ion-acoustic solitary waves in a space plasma are basically described by the nonlinear evolution equation, the periodic progressive waves and the algebraic solitary waves have not yet been investigated.

The object of this paper is to show the formation of relativistic periodic waves and the algebraic solitary waves in an unmagnetized space plasma with hot ions. In this paper, it will be expected that periodic progressive waves and algebraic solitary waves are obtained from a mixed modified K–dV (MK–dV) equation which includes the quadratic and the positive cubic nonlinearity. These waves are closely related to the periodic pulse of the potential concerning energetic solitary structure observed in the Earth's Magnetospheric space (Boström *et al.*, 1988, 1989).

The layout of this paper is as follows. In Section 2 we derive the MK–dV equation at the critical point where the nonlinear term of the K–dV equation can hardly contribute to forming the solitary wave. In addition, we derive a mixed MK–dV equation in the vicinity of the critical point from the basic equations associated with the fluid description of an unmagnetized collisionless relativistic plasma with a positive ion temperature. Section 3 is divided into two parts. We derive a relativistic periodic wave in Section 3.1 and a relativistic algebraic solitary wave in Section 3.2. The last section is devoted to the concluding discussion.

### 2. Derivation of a Mixed MK–dV Equation

We consider small but finite amplitude ion-acoustic waves propagating in a collisionless relativistic unmagnetized plasma. The dimensions of the system are much larger than the electron Debye length, and the plasma is quasi-neutral. Since the electron inertia and the electron trapping phenomena are neglected for low-frequency oscillations of ion-acoustic waves, the electron velocity is cancelled with the help of the Poisson's equation; the motions of electrons can be ignored. The fluid description used in this study can be justified by the fact that we are interested in the macroscopic, average nonlinear behaviour of the weakly relativistic plasma rather than the microscopic properties, that is, the motion of individual particles. This justification can be further supported by space and time-scales involved in this nonlinear problem. It is assumed that the velocity distribution of each species is Maxwellian everywhere and the weakly relativistic plasma is composed of a mixed fluid with hot and isothermal electrons and hot ion species. To simplify the discussion, we do not take into account kinetic effects such as the deviations from the Maxwellian distribution, Landau damping, etc. Since we consider the low-frequency motion of the ion-acoustic wave and assume that the longitudinal ion-acoustic wave propagates parallel to the electric field, we observe the electrostatic ion-acoustic wave in a weakly relativistic plasma. We assume that the ion flow velocity has a weakly