RESONANT TIDAL INTERACTIONS

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Abstract. The behaviour of stellar orbits is examined under the influence of a fixed triaxial potential and a tidal force. Changes in the kinetic energies in the principal directions are computed as a function of tidal interaction times; the important resonances are identified. Resonant interactions are discussed in relation to clusters of galaxies.

1. Introduction

Spitzer (1958) investigated the resonant interaction between a spherically-symmetric globular cluster and a passing point mass. The cluster was taken to exert linear forces in each of the three directions on the stars and the tidal force was assumed to result from a body passing at nearest distance of approach $p$ in a straight line with constant relative velocity $v$. As one might expect, when the typical interaction time $p/v$ is approximately equal to the period of oscillatory motion of a star in the unperturbed galaxy, there occurs a favourable condition for increasing the energy of stars. There is, of course, no reason why this type of argument should not be applied to the problem of interacting galaxies. If we consider elliptical galaxies, it would appear plausible because of their low ratio of rotational to dispersive velocities (see Illingworth, 1977), to view such objects as being tri-axial; Binney (1978), has argued that a non-spheroidal shape might be supported by pressure anisotropies. Schwarzschild (1979) has described a Monte-Carlo technique for building a self-consistent model. De Zeeuw and Merritt (1983) have investigated analytically the types of two-dimensional orbit in the rotational plane of a gravitational potential which is a combination of quadratic, bi-quadratic and quartic functions of two-dimensional coordinates. In his nonlinear analysis the bi-quadratic and quartic terms are taken to be perturbations to the quadratic ones and an asymptotic analysis is used. Any strongly nonlinear effects are exceedingly complicated to analyse; indeed even the frequencies of motion are functions of constantly changing amplitudes.

Binney (1980) has listed three possible ways of forming anisotropy dominated galaxies: (i) mergers, (ii) Lin et al. (1965) self-tidal distortion, and (iii) tidal interactions between galaxies. The latter mechanism while originally invoked as a source of angular momentum (Peebles, 1969) has been questioned by Efstathiou and Jones (1979) whose 1000-body dissipationless simulations produced little angular momentum a result which is not, however, at variance with the observations of elliptical galaxies. Binney points out that tidal effects might, however, still be effective in producing anisotropy energies. It would seem appropriate, within the context of interacting galaxies, to re-examine the resonant interaction between a passing point mass and stars moving in a three-dimensional quadratic potential, relaxing the assumption that the galaxy has a spherical...
potential, to determine the way in which the three different frequencies of oscillation combine to change kinetic energies or orbits in three principal directions of a triaxial system. The use of a quadratic potential is only a first approximation to the problem of interaction resonances. In a real galaxy the internal forces increase from the centre out a characteristic radius beyond which they decrease; this means that the frequencies of motion become dependent on amplitude in the sense that the greater the amplitude generally the lower the frequency and the more loosely bound are those stars.

2. Analysis

Consider a fixed gravitation potential $\phi$ of the form

$$\phi = \frac{(\alpha x)^2 + (\beta y)^2 + (\gamma z)^2}{2};$$

this being taken to be the potential due to the mass distribution in the galaxy. Added to the internal force, $-\nabla \phi$, let there be a further force imposed by a passing galaxy, of mass $M_p$, assumed to move rectilinearly in the $x-y$ plane with velocity $v$, parallel to the positive $y$-direction and passing within a distance $p$ of the positive $x$-axis. The equations of motion become

$$\ddot{x} + \beta^2 x = \frac{\lambda_0}{r^3} \left\{ 3 \frac{p v t}{r^2} y - \left( 1 - 3 \frac{p^2}{r^2} \right) x \right\},$$

$$\ddot{y} + \alpha^2 y = \frac{\lambda_0}{r^3} \left\{ 3 \frac{p v t}{r^2} x + \left( 2 - 3 \frac{p^2}{r^2} \right) y \right\},$$

$$\ddot{z} + \gamma^2 z = -\frac{\lambda_0}{r^3} z;$$

where a linear approximation to the tidal force is made relative to the centre of gravity $(x, y, z) = (0, 0, 0)$ of the main galaxy and $\lambda_0 = GM_p$ with $G$ the gravitational constant. The distance $r$ between the centres of gravity of the two galaxies is $\sqrt{p^2 + v^2 t^2}$ and time $t = -\infty$ is taken to be the initial time. It should be emphasized that the principal $x$-axis of the stationary potential $\phi$ is normal to the velocity of the passing galaxy.

At $t = \pm \infty$ the right-hand side of Equations (1)–(3) are zero and the motion is purely oscillatory with angular frequencies $\alpha$, $\beta$, and $\gamma$ in the $x$, $y$, and $z$-directions, respectively. We shall assume that $\lambda_0$ is small and calculate, to second order in $\lambda_0$, the change in $x$, $y$, $z$ due to the tidal force; the form assumed for displacement is $x = x_0 + \lambda_0 x_1 + \lambda_0^2 x_2 + O(\lambda_0^3)$ similarly for $y$ and $z$.

It is clear from the structure of the foregoing terms on the right-hand side of Equations (1)–(3) that the force has a typical life-time of approximately $2p/v$; thus it is intuitively reasonable to expect resonances when this figure is of the order of $2\pi/\alpha$ or $\beta$ or $\gamma$, i.e., when $p \approx v/\pi (\alpha^{-1}, \beta^{-1}, \text{or } \gamma^{-1})$. However, it is further clear that because the tidal forces