ETERNAL AND PULSATING UNIVERSES IN EINSTEIN–CARTAN SPACE-TIME

M. J. D. ASSAD and C. ROMERO
Departamento de Física, CCEN, Universidade Federal da Paraíba, João Pessoa, PB, Brazil

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Abstract. We exhibit a class of closed (Bianchi type IX), homogeneous and isotropic universes generated by an unpolarized spinning perfect fluid on the basis of Einstein–Cartan theory. The models are non-singular and pulsate around a static configuration in an endless number of cycles. All of them undergo periodic phases of accelerated (inflationary) expansion and may be reduced to the dynamics of an anharmonic oscillator.

1. Introduction

Although the Friedmann–Robertson–Walker (FRW) models are the most studied universes of the General Relativity (GR) theory, its evolutionary features are still scarcely known in the Einstein–Cartan (EC) theory (cf. Hehl et al., 1976). In spite of the fact that the flat FRW–EC models were already extensively discussed (cf. Trautman, 1973; Kopczynski, 1973; Kuchowicz, 1976; Gasperini, 1986), the evolution of the corresponding closed and open universes still deserves a deeper analysis. It was Kerlick (1975) who proved that one cannot incorporate a polarized perfect fluid (aligned distribution of spins) as source of a closed (Bianchi type IX) cosmology. Such a kind of model can only be generated by a perfect fluid in which the spins of its particles are randomly oriented. In this way, Kuchowicz (1978) exhibited FRW–EC exact solutions, among them a closed model which is non-singular and oscillates between a maximum and a minimum radius. However, he obtained this solution at the expense of getting rid of the spin conservation law in EC theory. The late assumption makes Kuchowicz’s particular model inconsistent with the field equations, as it will be seen. On the other hand, Nurgaliev and Ponomariev (1983) (NP) obtained a class of closed FRW–EC solutions, which are static and stable under homogeneous and isotropic infinitesimal perturbations.

Like the papers by Kuchowicz (1978) or Nurgaliev and Ponomariev (1983), we discuss the class of closed FRW–EC universes by assuming that the source of the gravitational field is an unpolarized spinning perfect fluid. By applying the methods of dynamical systems theory we will show that: (a) There exists a class of closed FRW–EC solutions which exhibit the essential features of Kuchowicz’s one, without the need of assuming the violation of the spin conservation law. (b) The class of static NP models is a special solution pictured in the phase space as an equilibrium point (a center). (c) The infinitesimally perturbed NP solutions appear in the phase portrait as elliptical orbits in the neighbourhood of the equilibrium point. (d) For finite perturbations of the static solution, the orbits are still closed, but no longer reproduce the harmonic behavior.
described by NP. (e) In both cases (infinitesimal and finite perturbations), the strong spin-spin repulsive effects overwhelms the gravitational attraction in the stages of huge density of matter, preventing the collapse to a singularity (the Universe reaches its minimum radius) and re-accelerating the Universe backwards to an expansion era. (f) Also, for large values of the expansion factor, the gravitational attraction would lead the Universe to recollapse (after reaching a maximum radius), since expansion weakens the spin terms. (g) The models evolve through cyclic inflationary epochs and one obtains such a scenario by either considering the source terms in the form of the postulated Weyssenhoff semi-classical energy momentum tensor (cf. Weyssenhoff and Raabe, 1947), as particularly done by Kuchowicz (1978) or Nurgaliev and Ponomariev (1983), or the Ray–Smalley improved energy momentum tensor (RS–EMT) of matter with spin (cf. Ray and Smalley, 1982, 1983). (h) By using the conformal form of the FRW metric the EC field equations reduce to the form of that an anharmonic oscillator: a simple model consistent with the phase portrait numerically integrated.

2. The Field Equations

We shall investigate the cosmological solutions of the EC field equations (units \( c = 8\pi G = 1 \))

\[
G_{AB} = R_{AB} - \frac{1}{2} g_{AB} R = T_{AB}^{\text{canonical}},
\]

\[
\mathcal{T}_{AB}^C = T_{AB}^C + 2\delta^C_{[A} T_{B]D}^D = S_{AB}^C,
\]

where \( G_{AB} \) is the Einstein tensor of the Riemann–Cartan space-time \( (U_4) \) and \( T_{AB}^{\text{canonical}} \) is the canonical energy momentum tensor for matter with spin. \( T_{AB}^C \) is the torsion, \( \mathcal{T}_{AB}^C \) is the modified torsion, and \( S_{AB}^C \) is the spin density tensor. Capital latin indices refer to a holonomic basis.

When one adopts the Weyssenhoff–Raabe (1947) semi-classical description for a perfect fluid with spin, as done in Kuchowicz (1978) or Nurgaliev and Ponomariev (1983), the spin density tensor is factorized in the form

\[
S_{AB}^C = S_{AB} u^C,
\]

\[
S_{AB}^A = S_{AB} u^A = 0,
\]

\[
S_{AB} = \delta_{ABCD} u^C S^D,
\]

where \( S_{AB} = -S_{BA} \), \( u^A \) is the normalized four-velocity of the fluid, \( S^D \) is the pseudovector of spin density and \( \delta_{0123} = 1 \). The phenomenologically assumed canonical energy-momentum tensor is given by

\[
T_{AB}^{\text{canonical}} = T_{AB}^F + 2\left( \mathcal{V}_C S_{AD}^C \right) u^D u_B,
\]

where \( \mathcal{V}_C \equiv \nabla_C + 2 T_{CD}^D \), and \( T_{AB}^F \) is the usual energy momentum tensor for a perfect fluid with energy density \( \rho \) and pressure \( P \)

\[
T_{AB}^F = (\rho + P) u_A u_B - P g_{AB}.
\]