Fourier analysis of the light curves of eclipsing variables, I

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Abstract. The aim of the present paper will be to pioneer a new approach to the analysis of the light changes of eclipsing binary systems in the frequency domain, and to point out its merits in comparison with a conventional treatment of the same problem in the time-domain which has been developed so far.

Following an introductory section in which the broad features of our problem will be set forth, Section 2 will contain an outline, and critique, of the time-domain approach. Section 3 will give an explicit treatment of the light changes arising from total and annular eclipses in the frequency domain -- a problem which we succeeded in solving in close algebraic form. Section 4 will extend this treatment to the case of partial eclipses; and in the concluding Section 5 the relative merits of our new results will be discussed in broader perspective.

Sections 3 and 4 contain explicit results pertaining to mutual eclipses of spherical stars exhibiting uniformly bright discs. An extension of these results to the case of arbitrary limb-darkening, and taking account of mutual distortion of both components, will be given in subsequent communications.

1. Introduction

A study of the light changes of eclipsing binary systems -- i.e., of close stellar couples which, by accident of the orientation of their orbits in space with respect to our line of sight happen to become eclipsing variables -- occupies a position of ever growing importance in contemporary astrophysics for several reasons. First, because of a prodigious abundance of such objects in the Universe around us -- both near and distant. A survey of the stars in the solar neighbourhood discloses that not less than 0.1% of them are components of binary systems exhibiting eclipses. If a similar proportion were characteristic of our Galaxy as a whole, the total number of eclipsing systems within it should be of the order of $10^8$, and probably closer to $10^9$. Eclipsing variables are, therefore, manifestly no exceptional or uncommon phenomena in the sky!

The significance of such systems is further underlined by the fact that they constitute also the only class of double stars whose binary nature can be recognized across great distances in space -- including external galaxies. In the neighbourhood of the Sun -- up to distances of a few hundred parsecs -- visual binaries can be identified by their orbital revolutions or (for very wide pairs) by common proper motions of their components. Spectroscopic binaries can be discovered (by their Doppler shifts) with modern large reflectors up to distances of several hundred (and, for intrinsically very bright systems, a few thousand) parsecs. Beyond this distance -- which means in more
remote parts of our Galaxy, as well as the entire realm of external galaxies down to
distances of the order of 10 million parsecs – double stars can be detected only if
they happen to be eclipsing variables.

Spectroscopic binaries which exhibit also photometric effects of eclipses have long
been our principal source of information on the masses and absolute dimensions
of individual stars – our only source for massive stars – of which only a minute frac-
tion of the total galactic supply has so far been utilized for this purpose. Even this
trickle has provided, however, the greater part of our present means to test the
theoretical mass-luminosity relations for the stars against empirical data; and when-
ever also the parallax of such systems is known (permitting us to convert the ap-
parent brightness of the respective object into its absolute luminosity), the observed
data lend themselves for a determination of effective temperatures of the stars of
different spectral types. A combination of the masses and absolute dimensions of the
components of eclipsing systems have provided material for studies of the chemical
composition (hydrogen content) of the stars, disclosing their evolutionary stage; and
under certain conditions (from apsidal motions) even of the degree of mass concen-
tration in their deep interiors. On the other hand, a distribution of light (described by
the limb- and gravity-darkening) over stellar discs undergoing eclipse – which can
likewise be deduced from photometric observations – can furnish valuable insight
into physical conditions prevalent in photospheric, and sub-photospheric, layers
of the respective stars. A study of eclipsing variables, which holds the clue to such a
range of astrophysical information (much of which cannot be obtained in any other
way) certainly needs no further motivation or apology!

The observational attention received by eclipsing variables has indeed been com-
mensurate with their numbers as well as importance; and accurate photoelectric
records of their light changes have been secured for several hundreds of such systems
(for their partial lists of recent date see, e.g., Koch et al., 1970; or Fracastoro, 1972).
Observations alone represent, however, only one aspect of the problem; the second
must be an interpretation of such data. In fact, the actual messages reaching us from
such systems from a distance too great to see them in the form of a two-dimensional
picture are expressed in terms of a ‘code’ which remains to be deciphered by appro-
priate methods.

The situation we face can be compared with the work of a television camera which
scans a scene and transforms what it sees sequentially, at high speed, into a series of
linear elements which are reassembled by the receiver into a two-dimensional picture.
An eclipsing binary represents, in principle, an analogous source of information;
for as one component proceeds to eclipse its mate, a scanning takes place, giving
rise to characteristic light changes that can be observed at a distance. Our variable
can, accordingly, be regarded as a rudimentary one-channel television system –
transmitting continuous light signals which a photometer records sequentially; and
the aim of the receiving set must be to reconstruct, from one-point time variation, a
time-independent two-dimensional picture of the system. Needless to say, the sim-