FINITE LARMOR RADIUS EFFECTS ON THE RAYLEIGH-TAYLOR INSTABILITY OF A ROTATING PLASMA OF VARIABLE DENSITY

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Abstract. The Rayleigh-Taylor instability in a rotating plasma of variable density has been investigated to include simultaneously the effects of viscosity and the finiteness of the ion Larmor radius. It is shown that, for a plasma in which the density is stratified along the vertical, the solution is characterized by a variational principle. Making use of this, proper solutions have been obtained for a semi-infinite plasma in which the density varies exponentially. The dispersion relation has been solved numerically and it is found that the influence of the effects of both FLR and viscosity is stabilizing. The Coriolis forces are found to have a dual role, stabilizing for small wave numbers and destabilizing for large wave numbers. The range of the small wave numbers, over which the Coriolis forces have a stabilizing influence, is found to increase with Coriolis forces.

1. Introduction

The problem of hydromagnetic stability of a magnetized plasma of variable density is of considerable importance in several astrophysical situations, e.g. in theories of sunspot magnetic fields, heating of the solar corona and the stability of stellar atmospheres in magnetic fields. Several authors have studied this problem, under varying assumptions, and a comprehensive account of the various investigations has been given by Chandrasekhar (1961).

Rosenbluth et al. (1962), Roberts and Taylor (1962), and Jukes (1964) have pointed out the importance of the effects of the finiteness of the ion Larmor radius, which exhibits itself in the form of a magnetic viscosity in the fluid equations, on plasma instabilities. Melchior and Popovich (1968) have included the FLR effect on the Kelvin-Helmholtz instability of a fully ionized plasma of variable density. Ariel and Bhatia (1969) studied the influence of the FLR effects on the Rayleigh-Taylor instability of a plasma in which the density has a one-dimensional gradient in a direction transverse to that of the static horizontal magnetic field. They found that the FLR effects have a stabilizing influence. The joint effects of Coriolis forces and FLR on this problem have also been studied by these authors (Ariel and Bhatia, 1970). Recently the first author (Bhatia, 1973) has investigated the combined influence of the FLR effects and kinematic viscosity on the problem of the Rayleigh-Taylor instability of a composite plasma of varying density.
It may, therefore, be of interest to study the problem of the Rayleigh-Taylor instability of a rotating plasma of variable density in which the effects of FLR (magnetic viscosity) and kinematic viscosity are included. This aspect forms the subject matter of this paper in which the direction of rotation is taken to be along that of the horizontal magnetic field. In the present paper, the effects of viscosity have been included by using the classical form of the Navier-Stokes equations. In view of this the results obtained may be taken to give only an insight into the tendencies of the actual physical situations.

2. Perturbation Equations

The linearized perturbation equations appropriate to the motion of an incompressible, infinitely conducting, viscous, rotating plasma are

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\begin{align*}
\frac{\partial \mathbf{u}}{\partial t} & = - \nabla \cdot \mathbf{P} + \frac{1}{4\pi} \left( \nabla \times \mathbf{h} \right) \times \mathbf{H} + \mathbf{g} \delta \mathbf{q} + 2\mathbf{q} \times (\mathbf{u} \times \mathbf{\Omega}) + \mu_0 \nabla^2 \mathbf{u}, \\
\frac{\partial}{\partial t} (\delta \mathbf{q}) + (\mathbf{u} \cdot \nabla) \mathbf{q} & = 0, \\
\frac{\partial \mathbf{h}}{\partial t} & = \text{curl} \left( \mathbf{u} \times \mathbf{H} \right), \\
\nabla \cdot \mathbf{u} & = 0, \quad \text{and} \quad \nabla \cdot \mathbf{h} = 0,
\end{align*}
\]

where \( \mathbf{u}(u, v, w) \), \( \mathbf{h}(h_x, h_y, h_z) \), \( \delta \mathbf{q} \) and \( \delta \mathbf{P} \) are the perturbations, respectively, in velocity, magnetic field \( \mathbf{H} \), density \( \mathbf{q} \) and stress tensor \( \mathbf{P}(P_{ij}) \) and \( \mathbf{g} = (0, 0, -g) \) is gravity, \( \mathbf{\Omega} \) is the uniform angular velocity with which the plasma is rotating and \( \mu_0 \) is the coefficient of viscosity (assumed to be constant). Taking the uniform horizontal magnetic field to be along the \( x \)-direction in a cartesian frame of reference, the components of the stress tensor \( P_{ij} \), taking into account the finite ion Larmor radius, are

\[
\begin{align*}
P_{xx} & = p, \\
P_{yy} & = p - qv \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right), \\
P_{zz} & = p + qv \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right), \\
P_{xy} = P_{yx} & = -2qv \frac{\partial u}{\partial z}, \\
P_{xz} = P_{zx} & = 2qy \frac{\partial u}{\partial y}, \\
P_{yz} = P_{zy} & = qv \left( \frac{\partial v}{\partial y} - \frac{\partial w}{\partial z} \right),
\end{align*}
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