WHY THE SUN MAY APPEAR OBLATE

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Abstract. If the Sun loses angular momentum from its core, due to core contraction, into the solar wind at the observed rate, then an \( \sim 0.7 \) day rotational period for the core of the Sun is required for temporal equilibrium. The rotational power released in the core contraction process can equal the observed magnetic energy released in the solar activity cycle if the Sun's core rotates with a period near 1.4 to 4 days. The rotational power released from a rotating object is \( \tau \Omega^2 \), where \( \tau \) is the torque on the object and \( \Omega \) is its angular velocity. Fitting this to the solar wind torque and core rotation rate provides an 0.5 to 5 day rotation period for the Sun's core. A gravitational Pannekoek-Rosseland electric field in the Sun makes the Ferraro theorem inapplicable in such a way that rather than a constant angular velocity with radius, an inverse square radial dependence occurs. This results in a two day rotational period for the region in the Sun where most of the angular momentum resides. The consistency of the above four methods suggests that the Sun's observed oblateness is due to a rapidly rotating solar core. The oblateness of the photosphere is estimated to be near \( 3.4 \times 10^{-6} \).

1. Introduction

Dicke and Goldenberg (1967, 1974) have observed the Sun's oblateness. Their best observations, in 1966, suggest a flattening of four and a half parts in one hundred thousand to an accuracy of one third part. This is distinctly different from the results obtained by heliometer measurements last century, discussed by Auwers (1891), where it was found the Sun was nearly round with some measurements suggesting a prolate appearance (see Ambronn, 1905; Poor, 1905; or Ashbrook, 1967 for a review). Although the earlier observations claim a greater accuracy than recent observations, Dicke and Goldenberg maintain that observer bias and atmospheric distortion were large sources of error not properly accounted for in their predecessors' work. The question of observer bias is always open to debate, but it is clear that the differential refraction of the atmosphere would place strong limits to the precision with which a heliometer could measure the distortion of the photosphere. Recent observations of Hill suggest a more nearly spherical appearance (see Physics Today 27, 17, 1974).

Theoretically, it is not understood why the Sun should be oblate. Although it is clear that a rapidly rotating solar core with a period of between \( \frac{1}{2} \) and 2 days could provide an explanation for the Sun's oblateness (see Dicke, 1970), it is not clear why the Sun's core should rotate so rapidly!

Strittmatter (1969) discusses many sides of the issue. Basically what has been done, in part, is to consider the Sun as rigid and calculate the spin-down time. A school of thought started by Howard et al. (1967) suggests the spin-down time is short due to angular momentum losses associated with thermally driven turbulence and 'Ekman' pumping (see articles by Goldreich and Schubert, 1967, 1968, as well as Dicke, 1970).
In the present paper, basically the rigidity of the Sun implied or assumed in the earlier works will be dropped. The well known ‘core contraction’ of stars will be seen to release rotational power and angular momentum. This, it is suggested, provides the basic driving mechanism to keep the Sun’s core rotating rapidly with a period near \( \frac{1}{2} \)–2 days as distinct from the surface 25-day rate.

1.1. SOLAR CORE CONTRACTION

Using Newton’s equations of motion in angular coordinates, we have

\[
\tau_{\text{s.,w.}} = \frac{d}{dt} (I\Omega) = \frac{dI}{dt} \Omega + I \frac{d\Omega}{dt},
\]

(1)

where the solar wind torque is the principal torque on the Sun being orders of magnitude greater than torques due to Doppler shifted photospheric radiation differences and where the parameters on the right can be referred to the Sun’s core where most of the mass, inertia and angular momentum reside.

In equation (1), previous efforts (see Howard et al., 1967) have neglected the changing moment of inertia with time and have shown that the second term leads to a small decay time for the Sun’s rotation rate. Thus the Sun appears to have reached whatever equilibrium it prefers. In this and subsequent sections, we neglect this second term (assuming this equilibrium) and compute only temporal changes in the Sun’s moment of inertia. In this section we use Equation (1) to calculate \( \Omega \) from \( \tau_{\text{s.,w.}} \) and \( \frac{dI}{dt} \), neglecting the second term on the right. We thus have

\[
\Omega = \frac{\tau_{\text{s.,w.}}}{\frac{dI}{dt}}.
\]

(2)

The solar wind torque may be estimated to be between \(-7 \) and \(-9 \times 10^{30} \) dyn cm \( \dot{\phi} \), from solar wind observations (Lazarus and Goldstein, 1971; Hundhausen, 1972a).

The reduction in the Sun’s moment of inertia may be obtained by considering the following physics. As material is being ‘burned’ in the solar core, the following reaction is generally thought to occur

\[
4H^+ + 4e^- \rightarrow 1\text{He}^{++} + 2e^- + n\nu + m \text{photons}.
\]

(3)

This reaction proceeds by a complex series of nuclear reactions. The above equation merely summarizes the main effect. When one complete nuclear reaction takes place, four hydrogen ions and electrons (eight ionized particles) are converted to one helium ion and two electrons (three ionized particles). Therefore, if the internal temperature is not changing rapidly, and no internal motions are allowed, a pressure decrease, \( \Delta P = \Delta N kT \), would occur because the radiation pressure is small. To allow this collapse, a slow contraction of the Sun’s core toward regions that are burning hydrogen should develop so as to conserve the number of particles per unit volume and therefore the pressure within the solar interior. The physics of this is well known as ‘core contrac-