FOURIER ANALYSIS OF THE LIGHT CURVES OF ECLIPSING VARIABLES, II

ZDENĚK KOPAL
Dept. of Astronomy, University of Manchester, England

(Received 8 October, 1974)

Abstract. The aim of the present paper will be to extend the Fourier methods of analysis of the light curves of eclipsing binaries, outlined in our previous communication (Kopal, 1975) in connection with systems whose components would appear as uniformly bright discs, to systems whose components exhibit discs characterized by an arbitrary radially-symmetrical distribution of brightness - i.e., an arbitrary 'law of darkening' towards the limb - be it linear or nonlinear.

In Section 2 which follows a few brief introductory remarks, fundamental equations will be set up which govern the light changes arising from the mutual eclipses of limb-darkened stars - be such eclipses total, partial or annular; and Section 3 will contain a closed algebraic solution for the elements of the occultation eclipses terminating in total phase. Such a solution proves to be no more complicated than it turned out to be for uniformly bright discs in our previous paper; and calls for no special functions for the purpose - as will be put in proper perspective in the concluding Section 4.

The cases of transit eclipses terminating in an annular phase, of partial eclipses of occultation or transit type, will be similarly dealt with by Fourier methods in the next paper of the present series.

1. Introduction

In a preceding communication recently published in this journal (Kopal, 1975; hereafter referred to as Paper I) a new method has been proposed for an analysis of the light changes of eclipsing binary systems in the frequency domain; and explicit procedure developed to this end for the simplest case of mutual eclipses of uniformly bright circular discs. This was done, not because the results based on so simplified assumptions could be readily applicable to actual observations (except under very special circumstances), but rather to demonstrate the new techniques on the simplest possible case. In order to make our method applicable to cases of greater astrophysical interest, the aim of the present paper will be to generalize the analysis given in Paper I to an interpretation of totally-eclipsing systems of stars whose apparent discs are characterized by an arbitrary radially-symmetrical distribution of brightness (i.e., are arbitrarily darkened towards the limb); though we shall continue to regard these discs as circular; and complications arising from their distortion will be postponed for the next communication.

2. Equations of the Problem

In order to do so, let us return to Equation (2.3) of Paper I which we shall now re-write as

\[ l = 1 - fL. \]  

\[ (2.1) \]
If, moreover, the distribution of surface brightness $J$ over the apparent disc of the star undergoing eclipse varies with the angle $\gamma$ of foreshortening in accordance with a law of the form

$$J = J_0(1 - u_1 - u_2 - \cdots - u_n + u_1 \cos \gamma + u_1 \cos^2 \gamma + \cdots + u_n \cos^n \gamma),$$

(2.2)

where $u_1, u_2, \ldots, u_n$ stand for the respective 'coefficients of limb-darkening', it can be shown (cf. Kopal, 1949; or 1959, Section IV.2) that

$$f(k,p) = \sum_{j=0}^{n} C^{(j)} \phi^j(k,p),$$

(2.3)

where the coefficients

$$C^{(0)} = \frac{1 - u_1 - u_2 - \cdots - u_n}{1 - \sum_{i=1}^{n} \frac{lu_i}{l + 2}}$$

(2.4)

and, for $j>0$,

$$C^{(j)} = \frac{u_j}{1 - \sum_{i=1}^{n} \frac{lu_i}{l + 2}}$$

(2.5)

while (cf. again Kopal, 1959; Section IV.5)

$$\pi r_1^{n+2} a_0 = 2 \int_{\delta - r_2}^{r_1} (r_1^2 - r^2)^{n/2} r \cos^{-1} \left( \frac{\delta^2 + r^2 - r_2^2}{2\delta r} \right) dr$$

(2.6)

if $\delta \geq r_2$, and

$$\pi r_1^{n+2} a_0 = \frac{2\pi r_1^{n+2}}{n + 2} - \int_{r_2 - \delta}^{r_1} (r_1^2 - r^2)^{n/2} r \cos^{-1} \left( \frac{r_2^2 - \delta^2 - r^2}{2\delta r} \right) dr$$

(2.7)

if $\delta \leq r_2$.

It should be added that the foregoing Equations (2.6) or (2.7) hold good as they stand only during partial phases of the eclipse – i.e., if

$$r_1 + r_2 \geq \delta \geq |r_1 - r_2|.$$  

(2.8)

Should $r_2 > r_1$ and $\delta < r_2 - r_1$, the eclipse becomes total, and Equation (2.7) reduces then to its first constant term

$$a_0 = \frac{2}{n + 2}.$$  

(2.9)

If, on the other hand, $r_1 > r_2$ and $\delta < r_1 - r_2$, our eclipse becomes annular – in which case Equations (2.6) and (2.7) should be replaced by