MAGNETO-RADIATIVE SHOCK WAVE PROPAGATION IN A
CONDUCTING PLASMA

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Abstract. Similarity solutions describing the flow of a perfect gas behind a spherical and cylindrical shock wave in a magnetic field with radiation heat flux have been investigated. The total energy of the expanding wave has been assumed to remain constant. The solutions, however, are only applicable to a gaseous medium where the undisturbed pressure falls as the inverse square of the distance from the line of explosion.

1. Introduction

The problem of propagation of shock wave in a non-homogeneous magnetic medium is of great interest in exploring the effect of explosion in the stars and atmosphere of the Earth.

The solution for cylindrically-symmetric flow has been obtained by Lin (1954). Ray (1957) has discussed the problems of point and line explosion and found an exact analytic solution. Analytic solutions in the three cases of plane, cylindrical, spherical flow have also been discussed by Sakurai (1955). Rogers (1958) has also studied the similarity solutions for three cases in uniform atmosphere. Recently, Singh and Vishwakarma (1984) have discussed the similarity solutions of the flow behind shock waves in a radiative magnetogas dynamic in which the total energy increases with time.

In the present paper the problem of explosion along a line in a gas cloud is discussed, similarity solutions are developed describing the propagation of a cylindrical and spherical shock in a non-uniform atmosphere with magnetic effect taking counter gas pressure and radiation heat flux into account. The radiation pressure and radiation energy are ignored. The gas in the undisturbed field is assumed to be at rest. We also assume the gas to be grey and opaque and the shock to be transparent and isothermal. The total energy of the explosion is constant.

2. Self-Similar Formulation

The equations for conservation of mass, momentum, energy, and equation of the magnetic field behind the wave are given by

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial r}(\rho u) + J\frac{pu}{r} = 0,$$

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\[
\frac{\partial}{\partial t} (pu) + \frac{\partial}{\partial r} (pu^2) + \frac{\partial p}{\partial r} + h + \frac{v h^2}{r} = 0, \quad (2)
\]

\[
\frac{\partial}{\partial t} (PE) + \frac{\partial}{\partial r} (puE) + \frac{\partial}{\partial r} (Pu) + \frac{1}{r^j} \frac{\partial}{\partial r} (qr^j) = 0, \quad (3)
\]

\[
\frac{\partial h}{\partial t} + \frac{\partial}{\partial r} (uh) + \frac{v hu}{r} = 0, \quad (4)
\]

where \( J = 1, 2 \) and \( v = 1, 1 \) for cylindrical and spherical, respectively; \( r \), the radial distance from the line of explosion; \( \rho \), density; \( P \), pressure; \( u \), radial velocity; \( q \), head flux; \( t \), time; \( E \), internal energy; and \( h \), magnetic field.

For an ideal gas

\[
E = \frac{P}{(\gamma - 1)\rho}, \quad P = \Gamma \rho T, \quad (5)
\]

where \( \gamma \) is the adiabatic gas index; \( T \), the temperature; and \( \Gamma \), the gas constant.

If we assume the local thermodynamic equilibrium and taking Rosseland’s diffusion approximation, we have

\[
q = -\frac{C \mu}{3} \frac{\partial}{\partial r} (\sigma T^4), \quad (6)
\]

where \( \frac{1}{4} \sigma C \) is the Stefan–Boltzmann constant; \( C \), the velocity of light; and \( \mu \), the mean-free path of radiation, is a function of density and temperature.

If we follow Wang (1966), we take

\[
\mu = \mu_0 \rho^x T^\beta, \quad (7)
\]

\( \mu_0, x, \) and \( \beta \) being constants. The disturbance is headed by an isothermal shock and the conditions are

\[
\rho_2 (V - u_2) = \rho_1 V = m_s, \quad (8)
\]

\[
P_2 - P_1 + \frac{h_2^2}{2} - \frac{h_1^2}{2} = m_s u_2, \quad (9)
\]

\[
E_2 + \frac{P_2}{\rho_2} \frac{1}{2} (V - u_2)^2 - \frac{q_2}{m_s} + \frac{h_2^2}{\rho_2} = E_1 + \frac{P_1}{\rho_1} \frac{1}{2} V^2 + \frac{h_1^2}{\rho_1}, \quad (10)
\]

\[
T_2 = T_1, \quad (11)
\]

where suffices 2 and 1 are for the region just behind and just ahead of the shock surface, respectively, and \( V \) denotes the shock velocity.

In front of the shock in the undisturbed gaseous medium, the density, pressure, and