RELATIVISTIC STATIC SPHERES FILLED WITH INFINITELY CONDUCTING CHARGED FLUIDS

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Abstract. A class of spherically-symmetric cosmological models for space-time filled with infinitely conducting relativistic perfect fluids with constant magnetic permeability is developed for a specified choice of matter density and fluid pressure. Some salient features of all these solutions are discussed.

1. Introduction

The problem of determination of exact solutions of Einstein field equations, Maxwell equations, and Einstein–Maxwell equations in general relativity has attracted a wide attention in recent researches.

Kramer et al. (1980) have discussed variety of solutions coupled with several techniques of generating new solutions concomitant with the above field equations.

The gravitational field in the exterior region of charged fluid spheres is described by Reissner–Nordstrom metric. Spherically-symmetric distributions of charged incoherent matter were studied by Bonnor (1960, 1965) and De and Raychaudhari (1968). Cooperstock and De la Cruz (1978) have shown that for charged perfect fluid spheres with non-zero pressure, in equilibrium $m^2 > q^2$, where $m$ denotes the gravitating mass and $q$ denotes the total charge contained within the sphere. Also Tikekar (1984) has discussed the formal features of Einstein–Maxwell equations for spherically-symmetric distributions of a charged perfect non-conducting fluids in equilibrium.

Lichnerowicz (1967) established the existence and uniqueness of solutions to Einstein equations for thermodynamical perfect fluid with infinite conductivity; which was later utilized by Yodzis (1971), Bray (1972), Shaha (1972), Date (1973), Mason (1977), Asgekar (1979), Suve and Asgekar (1979), in the subsequent development of the subject.

Maugin's (1972) matter field has been established with the help of action principle which encompasses the effects of magnetization and polarization of electromagnetic field on the internal structure of the relativistic magnetofluid. This scheme is used by Patil (1978) to derive some weak conservation laws and by Asgekar and Patwardhan (1988) to examine the Ferraro's laws of isorotation. The generalised form of Einstein static model and a class of spherically-symmetric non-static models for Maugin's magnetofluid are obtained by Asgekar and Aherkar (1984, 1988).
The prime goal of this paper is to propose appropriate static models for the Universe filled with Maugin's magnetofluid (i.e., infinitely conducting charged fluids) under the assumption of reasonable matter density and reasonable pressure. It is also shown that Reissner–Nordström metric and de Sitter metric are the members of this class.

Section 2 deals with Einstein field equations. Exact solutions of these field equations are found in Section 3. The dynamical features of the model are given in Section 4.

The four-dimensional space-time with the metric form
\[ ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta \]
is considered. The signature of the metric is taken as \((- , - , - , +\)). The Greek indices \( \alpha \) and \( \beta \) assume the values 1, 2, 3, 4. The summation convention on diagonally repeated indices is used throughout this paper. Semi-colons (\( ; \)) are employed to mean covariant differentiation. The units are such that the speed of light is unity. The overhead dash denotes differentiation with respect to \( r \).

2. Field Equations

The stress energy tensor for Maugin's (1972) magnetofluid under investigation is
\[ T^\alpha_\beta = (\rho + p + 2m)U^\alpha U_\beta - (\rho + 2m - \mu\delta^{\alpha}_\beta)g^\alpha_\beta - \mu H^\alpha H_\beta, \tag{2.1} \]
where \( \rho \) is the matter energy density, \( p \), the isotropic pressure of the fluid; \( \mu \), the constant magnetic permeability; \( U^\alpha \), the four-velocity of the fluid such that
\[ U^\alpha U_\alpha = 1, \tag{2.2} \]
\( H^\alpha \) is the space-like magnetic field vector such that
\[ H^\alpha H_\alpha = -\mu^2 \quad \text{and} \quad U^\alpha H_\alpha = 0 \tag{2.3} \]
and
\[ 2m = \mu h^2. \tag{2.4} \]

For the magnetofluid under investigation the field equations are well-known Einstein equations
\[ R^\alpha_\beta - \frac{1}{2} g^\alpha_\beta (R - 2\Lambda) = KT^\alpha_\beta, \tag{2.5} \]
and the Maxwell equations
\[ (U^\alpha H_\beta - U^\beta H_\alpha)_;_\beta = 0. \tag{2.6} \]

We consider the spherically-symmetric space-time metric in the Schwarzschild coordinates
\[ ds^2 = -e^\lambda dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + e^n dr^2, \tag{2.7} \]
where \( \lambda \) and \( n \) are functions of \( r \) alone, representing the geometry of the spherical Maugin's magnetofluid distribution in an equilibrium configuration.