THERMOSOLUTAL-CONVECTIVE INSTABILITY OF A COMPOSITE STELLAR ATMOSPHERE IN THE PRESENCE OF A VARIABLE MAGNETIC FIELD

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Abstract. The thermosolutal-convective instability of a composite stellar atmosphere is considered in the presence of variable horizontal magnetic field and collisional effects. The criteria for monotonic instability are obtained which generalize the criterion derived for thermal-convective instability in the absence of above effects.

1. Introduction

The convective instability (in which motions are driven by buoyancy forces) of a thermally unstable atmosphere has been termed as 'thermal-convective instability' by Defouw (1970). He has generalized the Schwarzschild criterion for convection to include departures from adiabatic motion and has shown that a thermally unstable atmosphere is also convectively unstable, irrespective of the atmospheric temperature gradient.

Defouw (1970) has established that the convective instability will set in if

\[ D' = \frac{1}{C_p} \left( L_T - \rho L_\rho \right) + \kappa k^2 < 0, \]

where \( L \) is the heat-loss function which depends only on its density and temperature; \( L_T \) and \( L_\rho \), partial derivatives of \( L \) with respect to temperature \( T \) and density \( \rho \) both evaluated in the equilibrium state. \( C_p \) is the specific heat at constant pressure; \( \kappa \), the thermal diffusivity; \( \alpha \), the coefficient of thermal expansion; and \( k \), the wave number of the perturbation.

The effects of a uniform rotation and a uniform magnetic field on thermal-convective instability of a stellar atmosphere have been studied separately by Defouw (1970) and simultaneously by Bhatia (1971). In astrophysical situations concerning stellar atmospheres, it has been found that the above inequality is a sufficient condition for monotonic instability in both cases. Quite frequently it happens that the plasma is not fully ionized but, instead, may be permeated with neutral atoms. Strömgren (1939) has reported that ionized hydrogen is limited to certain rather sharply bounded regions in space surrounding—for example, O-type stars and clusters of such stars—and that the gas outside these regions is essentially non-ionized. The medium has, therefore, been idealized as a composite mixture of a hydromagnetic (ionized) component and neutral component, the two interacting through mutual collisional (frictional) effects, by Hans (1968). The

thermal hydromagnetic instability of a partially-ionized plasma, for incompressible and compressible cases, has been studied by Sharma (1976) and Sharma and Misra (1986).

Usually the magnetic field has a stabilizing effect on the instability. However, Kent (1966) has studied the effect of a horizontal magnetic field which varies in the vertical direction on the stability of parallel flows and has shown that the system is unstable under certain conditions, while in the absence of magnetic field the system is known to be stable. In stellar atmospheres and interiors, the magnetic field may be variable and may altogether alter the nature of the instability. The Coriolis force also plays an important role on the stability of stellar atmospheres.

The conditions under which the convective motions are important in stellar atmospheres are generally far removed from the consideration of a single-component fluid and rigid boundaries. It is, therefore, desirable to consider a two-component fluid or one gas component acted on by a solute concentration gradient and free boundaries. In the case of a two-component fluid, buoyancy forces can arise not only from density differences due to variations in temperature but also due to variations in solute concentrations. The problem of the onset of thermal-convective instability is of great importance on account of its applications to atmospheric physics and astrophysics, especially in the study of ionosphere and outer layers of the solar atmosphere. Sharma and Sharma (1984) have studied the thermosolutal-convective instability in a stellar atmosphere.

In the present paper, we consider the thermosolutal-convective instability (the thermal-convective instability in the presence of stable solute gradient) of a composite stellar atmosphere in the presence of a variable horizontal magnetic field and collisional effects. The criteria for monotonic instability are obtained which generalize the criterion derived for thermal-convective instability in the absence of above effects.

2. Formulation of the Problem and Perturbation Equations

Consider an infinite horizontal composite layer consisting of finitely conducting hydromagnetic incompressible fluid of density \( \rho \) and a neutral gas of density \( \rho_d \), subjected to a stable solute concentration gradient and acted on by gravity force \( g(0,0,-g) \) and a variable horizontal magnetic field \( H(0,0,H_0(z)) \). This layer is heated from above such that a steady temperature gradient \( \beta = \frac{dT}{dz} \) is maintained. The layer is soluted from below such that a steady solute concentration gradient \( \beta' = \frac{dC}{dz} \) is maintained. If we regard the model under consideration we assume that both the ionized fluid and the neutral gas behave like continuum fluids and that the effects on the neutral component resulting from the presence of gravity and pressure are neglected. The magnetic field interacts with the ionized component only.

The first law of thermodynamics may be written as

\[
C_v \frac{dT}{dt} = -L + \frac{K}{\rho} \nabla^2 T + \frac{p}{\rho^2} \frac{d\rho}{dt},
\]