NONLINEAR GRAVIDYNAMICS:
ENERGY-MOMENTUM TENSOR OF COLLAPSAR FIELD

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Abstract. In the bounds of a theoretical scheme treating consistently gravitational interaction as dynamical
(gauge) field in flat space-time, an expression was obtained for the density of energy-momentum-tension of
gravitational field in vacuum around a collapsed object. A case was studied of an interacting static
spherically-symmetric field of a collapsar in vacuum with taking into account of input of all the possible
components (spin states of virtual gravitons) into the energy for the symmetric tensor of second rank \( \psi_{ik} \).
The radius of the sphere filled by matter for the collapsar of mass \( M \) may achieve values up to \( GM/c^2 \).

1. Introduction

This paper is the continuation of the paper (Sokolov, 1992) which began the study of
physical properties of objects with extremely strong gravitational fields on their surfaces,
i.e., on collapsars from the standpoint of consistent dynamical description of gravita-
tional interaction. The main purpose of the paper is the solution of the problem in
energy-momentum-tension of collapsar field when this gravitational field is strong.

In what follows, the question is basically one of the strong fields of objects with
masses of the order of several solar masses – i.e., on collapsars with stellar masses.
Macroscopic average densities of such objects are at least nuclear ones and achieve
supernuclear density. Correspondingly gravitational field energy densities on such
collapsars surface may become equal or even more than \( \rho_{\text{nuc}} c^2 \). If we take as an example
such object as neutron star with the mass \( M = 1.44 M_\odot \) and the radius 10 km and
evaluate the gravitational field energy density on the surface by the formula \( (\nabla \phi)^2 / 8 \pi G \),
then the energy connected with the field alone turns out to be enormous and approximates
to the rest energy of the matter of the neutron star itself.

May the field energy with the density greater or of the order of \( \rho_{\text{nuc}} c^2 \) be non-
localizable? In the bounds of purely geometrical (inertial) interpretation of gravitation
the answer to this question is known for a long time: the field energy is nonlocalizable
even in such a case. The consistent dynamical formulation of the theory of gravitational
interaction proceeds as a matter of fact from the notion that every cubic centimetre of
space contains a completely determined quantity of the energy-momentum-tension of
gravitational field. Of course a final answer in this debate will be obtained as a result
of the observations of space regions with strong field. Just in context of possible new
observational consequences I continue to formulate here the collapsar problem in
gravidynamics.

In particular, as was noted in our previous paper (Sokolov, 1992), the collapsars may

have surfaces. But for a rigorous proof of that, one must first of all clear up completely a question on the energy-momentum-tensor (EMT) of gravitational field.

In connection with the foregoing I shall emphasize through all the paper the characteristic features of the formulation of the collapsar field problem in direct connection with the field energy problem. For all this I consistently adhere to the theoretical scheme in which the gravitational interaction, equally with other ones, is considered as a dynamic field plunged in flat space-time. I note here once more that in such a case one may adopt at once that the energy is localizable, positive and is understood in the same sense as in any other field theory, in particular, in classical electrodynamics. I shall not prove especially the justice of such demands natural in dynamical field theory. It is more interesting to elucidate what observational consequences their fulfilment brings to.

We begin here (in the Introduction) with the most important, principle aspects laying on the basis of the approach developed by us (Sokolov, 1992; Sokolov and Baryshev, 1980; Baryshev and Sokolov, 1984) to the description of gravitational interaction. The term ‘gravitodynamics’ (GD) used below and also frequently used by gravitationists, seems to me the best one reflecting the features of our approach.

As was shown in detail in a previous paper (Sokolov, 1992), the field energy density near a gravitating body with the mass $M$ at the distance $r$ from its centre, can be given as a matter of fact by

$$
\theta_{\phi\phi} = \left(\nabla^2 \phi - \frac{2}{r} \phi \right) / 8\pi G,
$$

where $\phi = -GM/r$, if at its deduction one take into account the fulfilment of three conditions for the field EMT:

$$
\theta_{\phi\phi} \geq 0, \quad \theta_{\phi\phi} = \theta_{kl}, \quad \theta_{ik}\eta_{ik} = 0, \quad i, k = 0, 1, 2, 3;
$$

where $\eta_{ik} = \text{diag}(+1, -1, -1, -1)$ is Minkowsky’s metric tensor. I emphasize at once that in GD you may use only this always the same (constant) metric at consistent dynamic description of gravitational interaction so as you may do that in the case of all the other interactions.

To understand correctly following sections one should not forget and consistently adhere the concept (which became already a common place) that, in relativistic field theory, one may not ascribe straight away some finite dimension both to test particles and to particles – sources of field (‘matter’).

I.e., at formulation of relativistic theory of gravitational interaction (like all the modern field theories) it is more logical at least at the beginning to proceed from the fact that the right-hand sides of corresponding field equations can contain a point source or a system of point sources: i.e., a gravitating ‘body’. In particular, every macroscopic region, which a real gravitating body is formed of, can be presented as a pont with the mass $m_a$. These regions are ‘points’ between which mainly only gravitation acts.

In GD a fundamental special relativity conception of interacting points (usual for local field theory) is used as an initial concept of ‘gravitational charges’. Of course a question arises on the justice of such an idealized notion for macroscopic theory which is the gravitation theory. As we will see from the following, an exhaustive answer to such