AN INVERSE PROBLEM IN A THREE-DIMENSIONAL RADIATIVE TRANSFER

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Abstract. In a series of papers (cf. Bellman et al., 1965a, b; Kagiwada et al., 1975), an estimation of optical properties of turbid media has been made, in the least-squared sense, with the aid of quasi-linearization and invariant imbedding. Recently, an extension of the above procedure to the three-dimensional case with horizontally inhomogeneous albedo of the underlying surface has been attempted (Ueno, 1982). From computational aspects the numerical evaluation is not so easy, even by means of high-speed electronic computers. In the present paper it is shown that the latin-square algorithm is a useful estimate for the least-squares inference of the optical properties of turbid media bounded by an inhomogeneous reflecting surface.

1. Introduction

Recently, an estimation of unknown parameters in distributed parameter systems has been made from noisy measurements. In other words, in an inverse problem researchers tried to identify the coefficients of an operator with a known structure in terms of information yielded by some functionals of the solution. Such inverse problems, in which small perturbations in the observed functionals may result in large errors in the corresponding solutions are often called ill-posed in the classical sense (cf. Nashed, 1976). By imposing certain additional restrictions on the admissible solutions, ill-posed problems may possess stable solutions relative to data perturbations. Such problems are known as conditionally well-posed. A need for approximate solutions of conditionally well-posed problems with inaccurate data gave rise to an innovation, via regularization by which the identification problems are reduced to the minimization of performance (quadratic) functionals.

Putting particular emphasis on temperature sounding from an aspect of computational feasibility, several kinds of inversion methods such as analytical regularization by addition of solutions constraints, statistical regularization, iterated regularization and others have been discussed by several authors (cf. Deepak, 1977; Twomey, 1977; Fymat and Zuev, 1978). Furthermore, the mathematical inversion of the equations of radiative transfer is an important tool for the remote sensing and probing of the Earth-atmosphere system from the atmosphere and from space. In other words, the determination of the composition and structure of the atmosphere bounded by the ground (or ocean) is fundamentally important not only in operational and predictive meteorology but also
in the pattern recognition of digitized Earth imagery. In a series of papers by several authors (cf. Bellman et al., 1965a, b; Kagiwada, 1974, 1975; Ueno, 1982), applying an invariant imbedding and quasi-linearization to the minimization of the regularizing functional, an inverse problem of estimating the atmospheric optical thickness, an albedo for single scattering, the phase function and others has been numerically solved. In the present paper, assuming that the atmospheric optical properties are known, we show how to make a least-squares estimation of the horizontally inhomogeneous distribution of ground albedos from measured data with the aid of invariant embedding and the latin-square algorithm.

2. Radiative Transfer in a Three-Dimensional Slab

2.1. Basic Equations

In this section we deal with a three-dimensional radiative transfer model consisting of a free atmosphere bounded by a horizontally non-uniform reflector. Suppose that the top \( z = z_1 \) of a plane-parallel, vertically inhomogeneous, anisotropically scattering atmosphere of optical thickness \( \tau \) has mono-directional illumination \( \pi F \) per unit area normal to the direction of propagation. Let the intensity of radiation emergent in the direction \( \Omega \) from the level \( z \) (\( 0 \leq z \leq z_1 \)) at horizontal rectangular coordinates \( (x, y) \) be denoted by \( I(z, x, y; \Omega) \). In the above \( \Omega \) stands for \( (\theta = \cos^{-1} v, \phi) \), where \( \theta \) is a polar angle measure from the normal, and \( \phi \) is an azimuthal angle. Let the level-dependent phase function be denoted by \( p(z; \Omega, \Omega_0) \), in a manner similar to the level-dependent attenuation and scattering coefficients; i.e., \( \alpha(z) \) and \( \sigma(z) \). The equation of transfer appropriate to this case takes the form

\[
\nabla \cdot I(z, x, y; \Omega) + \alpha(z)I = \frac{\sigma(z)}{4\pi} \int p(z, \Omega, \Omega')I(z, x, y; \Omega') \, d\Omega',
\]

where \( -\infty < x, y < \infty \), \( d\Omega' = d\Omega' \, d\phi' \), and \( \nabla \cdot I \) is the directional derivative of \( I \) in the direction \( \Omega \). Equation (1) should be solved subject to the boundary condition

\[
I(z_1, x, y; +\Omega) = \pi F\delta(\Omega - \Omega_0),
\]

\[
I(0, x, y; -\Omega) = \frac{1}{v} \int k(x, y; \Omega, \Omega')I(0, x, y; +\Omega') \, d\Omega',
\]

where \( \delta(\Omega - \Omega_0) = \delta(v - u)\delta(\phi - \phi_0) \). In Equation (3) the bidirectional reflectance \( k(x, y; \Omega, \Omega') \) represents the probability that a photon incident on the bottom \( (0, x, y) \) in the direction \( \Omega' \) will be reflected from it in the direction \( \Omega \) within an elementary solid angle. In the case of isotropic reflection we put

\[
k(x, y; \Omega, \Omega') = uA(x, y)/\pi,
\]