DENSITY PERTURBATIONS IN THE BRANS–DICKE THEORY

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(Received 4 April, 1996; accepted 26 February, 1997)

Abstract. We give here the calculation of density perturbations in a gravitation theory with a scalar field non-minimally coupled to gravity, i.e., the Brans–Dicke theory of gravitation. The purpose is to show the influence of this scalar field on the dynamic behaviour of density perturbations along the eras where the equation of state for the matter can be put under the form $p = \alpha \rho$, where $\alpha$ is a constant. We analyse the asymptotic behaviour of these perturbations for the cases $\alpha = 0$, $\alpha = -1$, $\alpha = 1/3$ and $\rho = 0$. In general, we obtain a decaying and growing modes. In the very important case of inflation, $\alpha = -1$, there is no density perturbation, as it is well known. In the vacuum phase the perturbations on the scalar field and the gravitational field present growing modes at the beginning of the expansion and decaying modes at the end of this phase. In the case $\alpha = 0$ it is possible, for some negative values of $\omega$, to have an amplification of the perturbations with a superluminal expansion of the scale factor. We can also obtain strong growing modes for the density contrast for the case where there is a contraction phase which can have physical interest in some primordial era.

1. Introduction

The existence of a classical scalar field in Nature has been considered in many theories of gravitation that present alternatives to General Relativity. The prototype of scalar theories is the Brans–Dicke Theory [1], [2], [3], [4], whose Lagrangian is given by:

$$S = \frac{1}{16\pi} \int d^4 x \sqrt{-g} \left[ \phi R - \omega \left( \frac{\phi \mu \phi^{\mu}}{\phi} \right) + 16\pi L_{\text{mat}} \right].$$

(1)

It can be expected that the presence of the scalar field leads to different predictions with respect to those we obtain in General Relativity. In cosmology, this can lead to far reaching consequences, since the standard scenario given by General Relativity presents, besides some spectacular success, important drawbacks such as the horizon, flatness and structure formation problems [5].

However, local physics limits the value of the parameter $\omega$: it must be greater than 500 to account the classical tests. This result has reduced the interest in the Brans–Dicke Theory. This situation has changed recently with the proposal of the extended inflation [6], [7]; in the de Sitter phase, Brans–Dicke Theory predicts power-law inflation instead of exponential; in fact, following D. La and P. Steinhardt, at the beginning of inflation the BD solutions (for $p = -\rho$) approaches the Einstein–de...
Sitter solution. In the second stage of the inflation, both the scalar field and the scale factor grow by power law rather than exponential. This feature prevents the so-called ‘graceful exit’ problem. However, in order to work, the parameter must be $\omega \approx 24$, contradicting observation; nevertheless this constraint follows from local conditions. This drawback can be overcome through a generalization of the original Brans–Dicke Theory, allowing the parameter $\omega$ to be a function of the field $\phi$ itself.

This revival of Brans–Dicke Theory leads us to ask if the problem of structure formation can be modified through the introduction of scalar field. This question has been treated in many different situations in the literature [8], [9], [10]. Here, we propose to study the evolution of density fluctuation in the traditional Brans–Dicke Theory in the different phases of the Universe. Even if we consider $\omega$ as a constant, this analysis can furnish many insights in how the scalar field modifies the main conclusions about gravitational instability obtained employing General Relativity. In particular we will see that in de Sitter phase, the scenario differs substantially from the traditional one. In the other phases, however, the differences are much less important. As was mentioned above, the main interest in the presence of the scalar field is the possibility that this field accelerates the growth of density perturbations.

We will work in the Lifschitz-Khalatnikov formalism [11]. The reason is that it furnishes essentially the same relevant results we can obtain employing another formalism, and also it allows for the choice of synchronous frames where the physical interpretation of the concerned quantities are easily done. We must, of course, be careful about the presence of unphysical modes [12]. However they can be eliminated by performing an infinitesimal coordinate transformation. Taking care of the so called residual coordinate freedom, we can be sure to retain just the physical modes.

We will allow for negative values for the parameter $\omega$. In fact, a negative $\omega$ is what is predicted by the effective models coming from Kaluza-Klein and Superstring theories [13].

This paper is organized as follows. In section II we present the background solutions of the unperturbed universe in the following phases of its development: vacuum, inflation, radiation and incoherent matter. In section III we obtain the perturbed equations and their solutions in terms of Bessel functions for that phases. In section IV we calculate the asymptotic behaviour of the solutions for $t \to 0$ and $t \to \infty$. Finally, in section V we discuss the relations between the wavelength $\lambda$ and the particle-horizon distance $H^{-1}$.

We assume in this article the following notations: the greek indices run from zero to three; the latin indices run from one to three; the signature is $(+, -, -, -)$; we use a Robertson-Walker metric with flat space section $(k = 0)$; the scalar field $\phi$ is a time function; the energy-momentum is the perfect fluid; we use the synchronous gauge which fix the reference frame.